FIRST QUARTERLY REPORT

(DESIGN STUDY REPORT)

FOR

A DAY-NIGHT HIGH RESOLUTION INFRARED RADIOMETER EMPLOYING TWO-STAGE RADIANT COOLING

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VOLUME II OF TWO VOLUMES

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APPENDIX I

OPTIMIZATION OF COOLER GEOMETRY

The optimization of the cooler geometry determines the relative dimensions and the angles in the vertical and horizontal planes of each stage. The optimization is carried out by minimizing the ratio of cone length (distance from a radiant patch of zero thickness along the patch normal to the cone mouth) to patch half width in the vertical and horizontal planes, subject to the constraint that the patch must have a maximum look angle. This procedure places a given patch area in a cooler of minimum length or, conversely, a maximum patch area in a cone of given length. It determines the cone angles in each plane and the aspect ratio of the rectangular radiant patch.

The geometry of a radiant cooler stage through a vertical or horizontal plane is shown in Figure I-1. This may also be considered a section of a truncated right circular cooler. The cone length ℓ of the stage is given by

$$\ell = (r_2 - r_1) \cos \theta \qquad \text{(I-1)}$$

where

r_2 = distance along cone surface from apex to mouth

r_1 = distance along cone surface from apex to patch

θ = half angle of cone

The total length of a two-stage cooler, from cone mouth to cone mouth, is 2ℓ. The half width of the patch in the plane is given by

$$c = r_1 \sin \theta \qquad \text{(I-2)}$$

And the ratio of cone length to patch half width is

$$\frac{\ell}{c} = \frac{1 - \dfrac{r_1}{r_2}}{\dfrac{r_1}{r_2}} \cot \theta \qquad \text{(I-3)}$$

Now the maximum look angle, ϕ, for a spherical patch is given by (See Figure I-2)

$$\sin (\phi - \theta) = \frac{r_1}{r_2} \qquad \text{(I-4)}$$

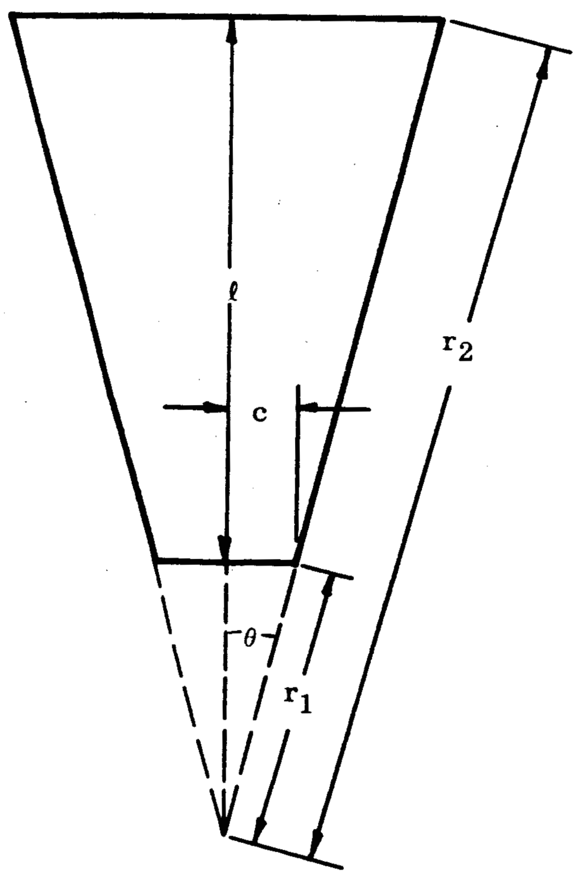

Figure I-1 Radiant Cooler Geometry

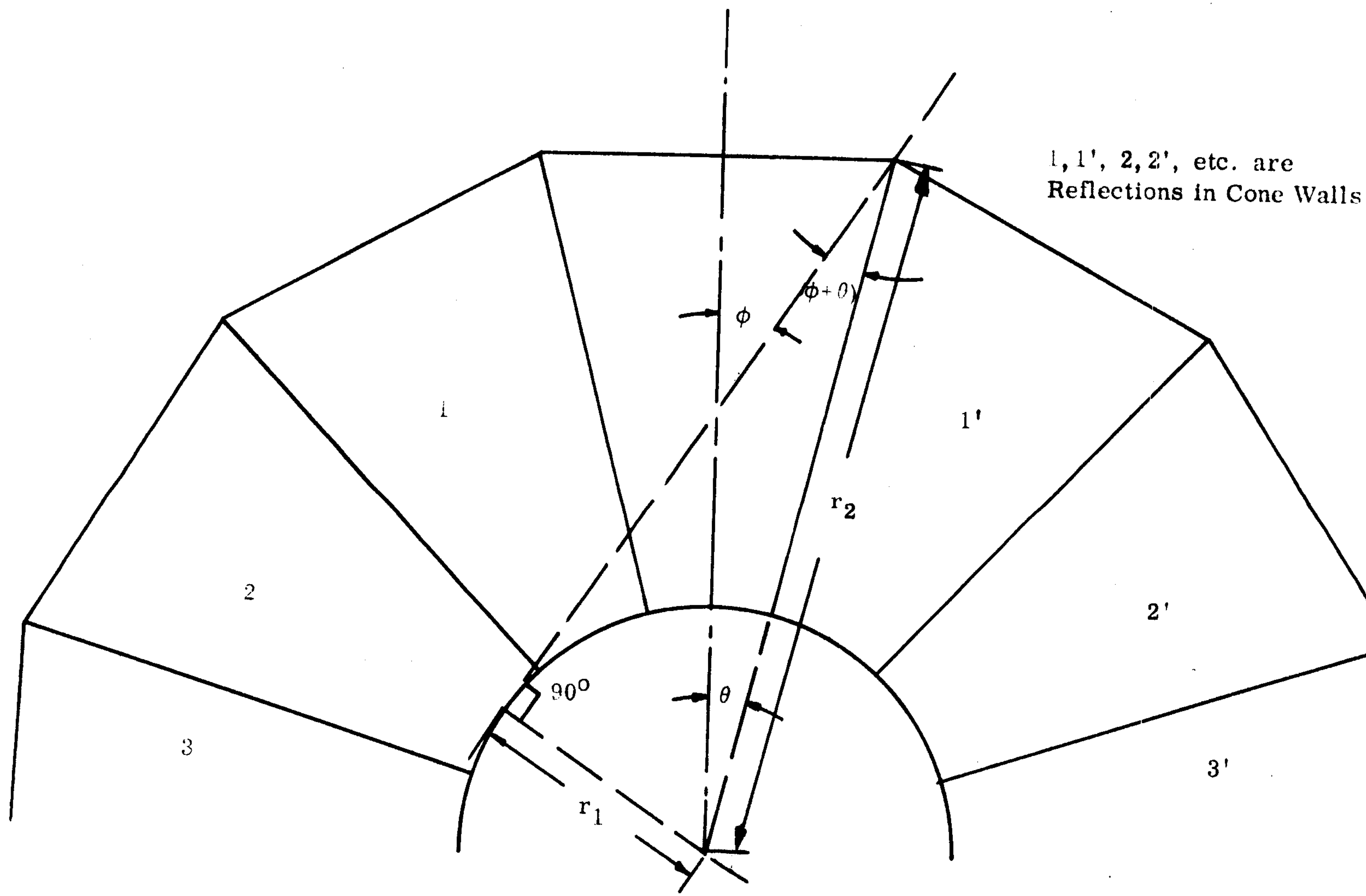

Figure I-2 Maximum Look Angle ϕ For a Spherical Patch

This relation is a good approximation for a flat patch when θ is small and a safe value of any θ, since ϕ for a flat patch is always less than or equal to ϕ for a spherical patch in a cone of the same geometry (same θ and r_1/r_2). The angle ϕ is the maximum angle to the patch normal (axis of the cone) at which radiation from the patch leaves the cone mouth and the maximum angle to the cone axis of an external object that can be seen from the patch area. The effect of the cone is therefore to act as a crude collimator for patch radiation and as a set of blinders to an observer on the radiant patch.

Thus for a given maximum look angle

$$\frac{\ell}{c} = \frac{1 - \sin(\phi - \theta)}{\sin(\phi - \theta)} \cot\theta \qquad \text{(I-5)}$$

The cone length to patch half width ratio for fixed ϕ is then only a function of the cone half-angle θ.

To minimize ℓ/c,

$$\frac{d(\ell/c)}{d\theta} = 0 = \cot\theta \cdot \cos(\phi - \theta)$$

$$- \csc^2\theta\,[1 - \sin(\phi - \theta)] \cdot \sin(\phi - \theta) \qquad \text{(I-6)}$$

For small cone angles we may use the approximation $\cos\theta = 1$, in which case (I-6) can be solved for $\sin\theta$. The result is

$$\sin\theta = \frac{-\cos\phi\,(1 - \sin\phi) + (1 - \sin\phi)^{1/2}}{\cos^2\phi + \sin\phi} \qquad \text{(I-7)}$$

This formula can be used to determine a first approximation to the optimum cone half angle, which can be used as the starting value in equation (I-5) to determine a more accurate result.

Once the optimum value of the cone half angle has been calculated, the ratio r_1/r_2 is set by equation (I-4) for the given maximum look angle. This sets the geometry, but not the scale, of the cooler. The ratio of minimum ℓ/c values in the horizontal and vertical planes yields the optimum aspect ratio of the radiant patch.

Plots of ℓ/c versus θ are given in Figure I-3 for maximum patch look angles of 30 degrees, 31.5 degrees, 45 degrees, and 60 degrees. Note that $\ell/c \longrightarrow \infty$ as $\theta \longrightarrow \phi$ (equation I-5) and that the curve is broader for larger values of ϕ. The values of optimum θ and minimum ℓ/c are given in Table I-1 and the optimum patch aspect ratios for various combinations of vertical and horizontal look angles in Table I-2.

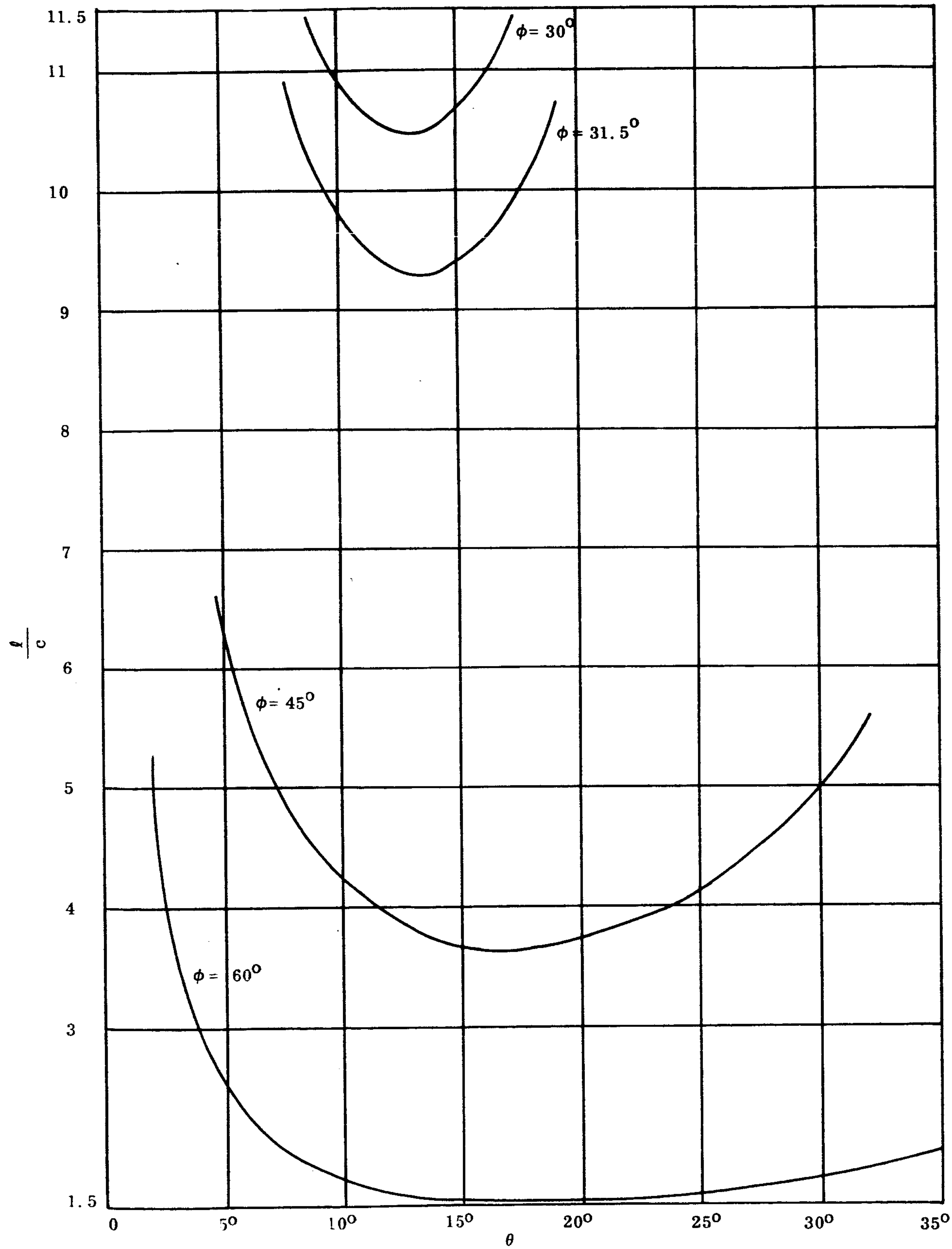

Figure I-3 Relative Length Versus Cone Half Angle

Table I-1

Optimum Cone Geometry for Given Maximum Look Angle

ϕ	θ_{opt}	(ℓ/c) min
30^o	13^o	10.48
31.5^o	13.5^o	9.314
45^o	17^o	3.69
60^o	18^o	1.520

Table I-2

Optimum Patch Aspect Ratios for Given Maximum Look Angles

Vertical	Horizontal	Aspect Ratio
30^o	60^o	6.89
31.5^o	60^o	6.13
45^o	60^o	2.43
45^o	45^o	1.00
60^o	60^o	1.00

There are other optimizations possible in addition to the one carried out above to minimize the cone length to patch half width ratio. One would like to find an optimization procedure directly related to the thermal performance, for example. This might be accomplished by minimizing the radiative coupling between the patch and its cone (Appendix II). The geometrical parameters which influence radiative coupling are θ and ϕ, and for a fixed ϕ the minimum occurs at $\theta = \phi$. This means $r_1/r_2 \longrightarrow 0$ and $\ell/c \longrightarrow \infty$, that is, either an infinitesimal patch or an infinitely long cone is required. Neither solution is practical, of course, and this approach to optimization was abandoned.

APPENDIX II

EMISSIVITY OF GOLD SURFACES IN SINGLE-STAGE RADIANT COOLER

Equations are derived from the effective emissivity and effective reflectivity of the cone structure as viewed from the patch in terms of the actual surface emissivity. These equations are then used to estimate the average surface emissivity of the gold-coated surfaces in the single-stage, 200 degrees K radiant cooler from performance data using a right circular equivalent to the rectangular cone structure. These equations are also used in Appendix III to calculate the effective patch to cone emissivity of both stages of the two-stage cooler using the maximum gold surface emissivity in the single-stage cooler and the emissivity of a good evaporated gold surface.

<u>Calculation of Effective Emissivity from Surface Emissivity</u>

The black radiating patch in a radiant cooler views cold space via a low-emissivity, high-reflectance cone structure. If f_n is the fraction of radiant flux from the patch which reaches space after n reflections from the cone walls, the effective reflectance of the cone, as seen from the patch, is

$$\rho_{pc} = \sum_{n=0} f_n \rho_g^n = \sum_{n=0} f_n (1 - \epsilon_g)^n \qquad \text{(II-1)}$$

where ρ_g is the reflectance of the cone surface and ϵ_g its emissivity. The effective reflectance of the cone is the fraction of radiant power emitted by the patch that reaches cold space. This concept is used in the study of cavities to express the fraction of incident radiation reflected back out of the cavity (See, for example, E. W. Treuenfels, JOSA 93, 1162, 1963). In the case of the radiant cooler, the cavity is in the form of a truncated conical perforation.

According to Kirchhoff 's radiation law, the effective emissivity, ϵ_{pc}, of the cone structure, as seen from the patch, is then $(1 - \rho_{pc})$. That is,

$$\epsilon_{pc} = 1 - \sum_{n=0} f_n (1 - \epsilon_g)^n \qquad \text{(II-2)}$$

This is one of the three general methods employed for deriving equations for cavity emissivity (G. A. Rutgers, Handbuch der Physik, Bd. 26 (Springer-Verlag, Berlin, 1958), p. 129).

An expression for the effective patch to cone emissivity may also be obtained directly with the help of Figure II-1. ABCD is a cross-section of a radiant cooler in which BC is the black patch. BC' and CB' are images of the patch formed by one reflection in the cone walls; C'B'' and B'C'' are images of the patch by two reflections. The cone walls are assumed to be specularly

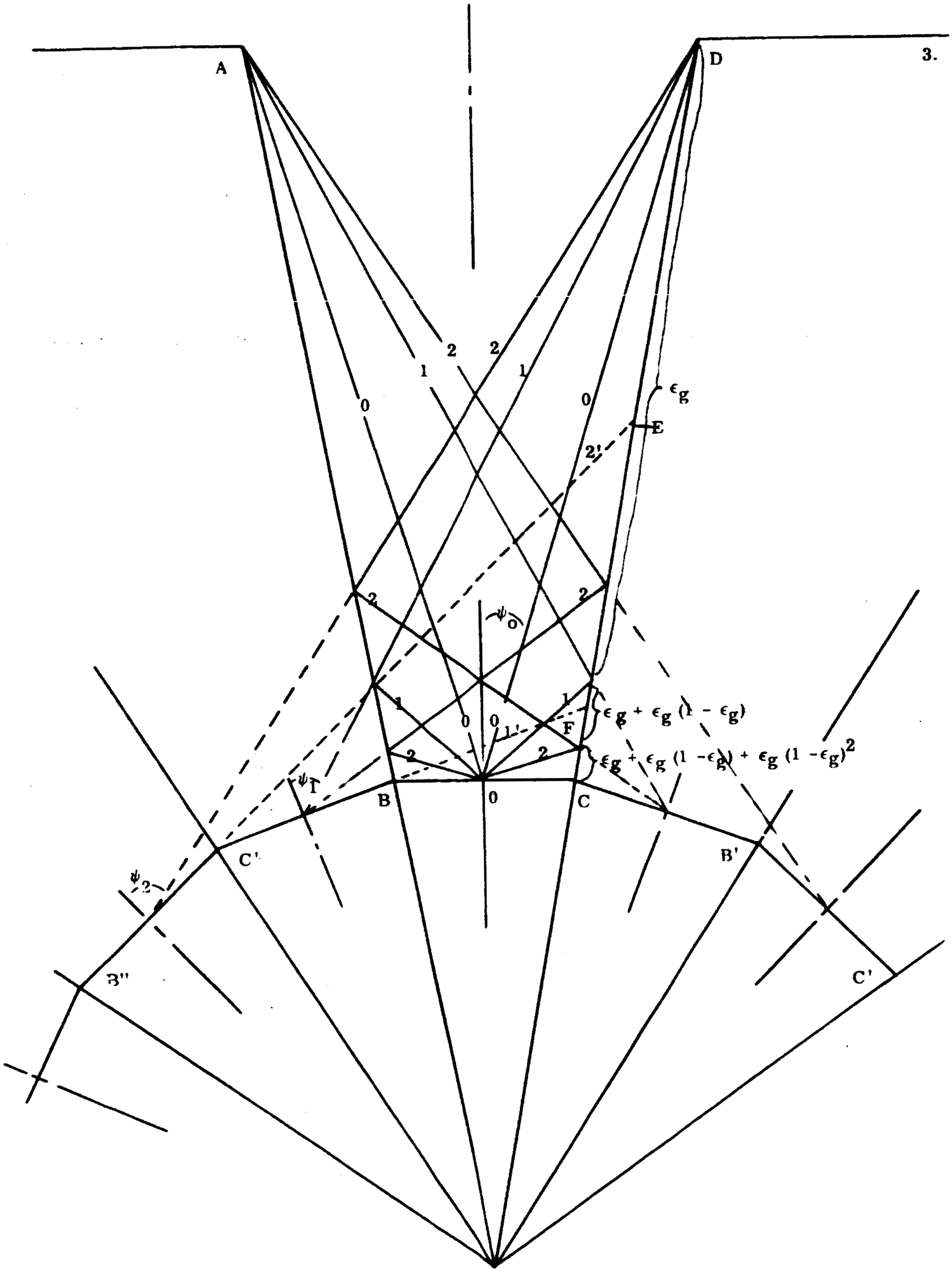

Figure II-1 Calculating of Effective Patch to Cone Sensitivity reflecting and the patch to have an absorptance of unity. The technique of multiple-reflections shown in the sketch has been used by Sparrow and Lin to determine the emissivity of a specularly reflecting V-groove cavity ("Absorption of Thermal Radiation in V-Groove Cavities", U. of Minn. for NASA-Lewis, N62 10682, July 1962). It has also been employed by Williamson (JOSA 42, 712, 1952) and Hanel (ARS Journ. 31, 246, 1961) in the study of specularly reflecting cone channels.

The view of the cold space and the cone walls from the patch will be approximated by the view from the center of the patch. The fraction, f_o, of radiation emitted by the patch between the normal to the patch and the rays 0 goes directly to space without reflection from the cone and is equal to F_{ps}, the view factor from the patch to space (i. e., the fraction of emitted radiation going directly to space). Rays between 0 and ray 1 are reflected once from the cone wall before going to space; rays between 1 and 2 are reflected twice and between 2 and 90 degrees to the patch normal, three times. The fraction of patch radiant flux which reaches cold space is then

$$\rho_{pc} = f_o + f_1 \rho_g + f_2 \rho_g^2 + f_3 \rho_g^3 + \cdots = \sum_{n=0} f_n \rho_g^n \qquad \text{(II-3)}$$

The radiant flux from the patch which is absorbed by the cone wall is then given by

$$\Phi_{p-c} = \sigma T_p^4 A_p (1 - \rho_{pc}) \qquad \text{(II-4)}$$

where T_p is the patch temperature and A_p the patch radiating area. The radiant flux from the patch therefore either goes to cold space or is absorbed in the cone walls. None is returned to the patch because of the outward sloping, specularly reflecting walls.

When reversed in direction, the paths by which rays go to space via reflections from the cone walls become the paths by which radiation from the cone walls reaches the patch. We may therefore expect that since $(1 - \rho_{pc})$ is the effective absorptance for radiation going from the patch to space, that it is also the effective emissivity for radiant transfer from the cone walls to the patch. This is shown by means of Figure II-1.

Radiation from the cone wall may reach the patch by a direct path or by reflection off the cone wall. All the cone wall can see the patch directly. The cone wall between D and F can see the patch by one reflection off the walls; this area is determined by the rays 1 and 1' from the image of the patch formed by one reflection. This once-reflected cone radiation may be accounted for by assigning an emissivity $\epsilon_g (1 - \epsilon_g)$, in addition to the emissivity ϵ_g due to direct coupling, to the cone wall area between 90 degrees to the patch normal and the boundary where ray 1 from the patch center first strikes the wall. The once-reflected wall radiation effectively comes from this area, since it is reflected from the area before being absorbed by the patch. Similarly, the cone wall between D and E can see the patch by two reflections in the cone wall; this area is determined by rays 2 and 2' from the image of the patch formed by two reflections. The twice reflected radiation may be accounted for by assigning a third emissivity component of $\epsilon_g (1 - \epsilon_g)^2$ to the cone wall area between 90 degrees to the patch normal and the boundary where ray 2 from the patch center first strikes the cone wall.

The above procedure may be extended until a reflection m is reached such that the mth reflection of the patch in the cone walls cannot see the cone walls. In the attached sketch, the maximum number of wall reflections for wall emission is two, so that m is three. The radiant power transfer from the cone wall to the patch can therefore be determined by assigning an emissivity

$$\epsilon_n = \epsilon_g \; [1 + (1 - \epsilon_g) + \cdots + (1 - \epsilon_g)^{n-1}] \qquad \text{(II-5)}$$

to the wall area A_n, where A_n is the wall area initially intercepted by rays from the patch center that require n reflections to go to cold space. The above is a geometrical progression and may be summed to give

$$\epsilon_n = [\, 1 - (1 - \epsilon_g)^n \,] \qquad \text{(II-6)}$$

The radiant power transfer from the cone walls to the black patch is then

$$\Phi_{c-p} = \sigma \, T_c^{\,4} \sum_{n=0} A_n \, g_n \; [1 - (1 - \epsilon_g)^n] \qquad \text{(II-7)}$$

where g_n is the view factor from A_n to the patch area A_p (i.e., the fraction of diffuse radiation from A_n which goes directly to A_p). But

$$A_n \, g_n = A_p \, f_n \qquad \text{(II-8)}$$

where f_n is the view factor from the patch to area A_n (M. Jakob, Heat Transfer, Vol. II, Wiley and Chapman & Hall, 1957, p. 9). The view factor f_n is equal to the fraction of patch radiation that requires n reflections in a perfectly reflecting cone ($\rho_g = 1$) to reach cold space. Hence,

$$\Phi_{c-p} = \sigma \, T_c^{\,4} \, A_p \sum_{n=0} f_n \, [1 - (1 - \epsilon_g)^n] \qquad \text{(II-9)}$$

The sum over n may be rewritten $\sum_{n=0} f_n - \sum_{n=0} f_n \, (1 - \epsilon_g)^n$. But

$$\sum_{n=0} f_n = 1 \qquad \text{(II-10)}$$

(Jakob, op. cit., p. 10), that is, the sum of all view factors from the patch (over a hemisphere) is unity. Equation (II-9) then becomes

$$\Phi_{c-p} = \sigma T_c^4 A_p \quad [1 - \sum_{n=0} f_n (1 - \epsilon_g)^n] \tag{II-11}$$

Finally, since the patch is black (absorptance of unity), no wall radiation is reflected back to the walls, and the net radiative exchange between the cone walls and the patch is (Equation II-4 and II-11)

$$\Delta \Phi_{c-p} = \Phi_{c-p} - \Phi_{p-c} = \sigma A_p (T_c^4 - T_p^4) \; [1 - \sum_{n=0} f_n (1 - \epsilon_g)^n] \tag{II-12}$$

The effective emissivity for cone wall-patch radiant exchange is therefore

$$\epsilon_{pc} = (1 - \rho_{pc}) = 1 - \sum_{n=0} f_n (1 - \epsilon_g)^n \tag{II-13}$$

as already obtained by application of Kirchhoff's law. For $\epsilon_g^2 < 1$

$$(1 - \epsilon_g)^n = 1 - n\,\epsilon_g + \frac{n\,(n-1)\,\epsilon_g^2}{2} - \cdots \tag{II-14}$$

and for $\epsilon_g \ll 2/(n-1)$ i.e., $\epsilon_g \ll 1$ for $n \leq 3$

$$(1 - \epsilon_g)^n = 1 - n\,\epsilon_g \tag{II-15}$$

The equation for the effective patch to cone emissivity then becomes

$$\epsilon_{pc} = \epsilon_g \sum_{n=0} n\, f_n \tag{II-16}$$

Average Surface Emissivity of the Single-Stage Cooler

The performance of the single-stage cooler may be expressed by its thermal balance equation

$$H_i \, (T_s^4 - T_p^4) + K_c \, (T_s - T_p) = \rho_c \, \sigma \, A_r \, T_p^4 \tag{II-17}$$

where

T_s = temperature of surroundings

K_c = conductive transfer coefficient

H_i = radiative transfer coefficient

In the single-stage 200 degrees K radiant cooler, no more than three reflections off the cone wall are necessary for a ray to go from the center of the radiating patch area to cold space. Equation (II-16) for the effective cone emissivity transfer can therefore be used, and the equation for the radiative coefficient becomes

$$H_i = \sigma \epsilon_g \left[A_r \sum_{n=0} n f_n + 1/2\, A_b\right] \qquad \text{(II-18)}$$

where

A_r = radiating (front) patch area.

A_b = non-radiating (back) patch area plus other surfaces radiantly coupled to cylinder

ϵ_g = average surface emissivity

A_r is 1.52 in^2 and A_b, 3.04 in^2; ϵ_g is the average among the cone walls, the area A_b, and the cylinder structure surrounding A_b. The equation assumes plane-parallel geometry or its equivalent between A_b and a specularly reflecting cylinder structure. From data taken during test runs on HRIR model F-3, T_p = 201 degrees K when T_s = 290 degrees K. The conductive coefficient for the six chromel AA suspension wires is 0.278 mw/degrees K. Equation (II-17) then becomes, on substituting the values for A_r and A_b and solving for H_i.

$$H_i = \frac{(65.9 - 90.6\, \epsilon_g\, \Sigma\, n\, f_n)}{54.4} \times 10^{-8} \text{ mw}/(^{\circ}\text{K})^4 \qquad \text{(II-19)}$$

Equating this to equation (II-18), ϵ_g can be determined if the f_n are known.

The view factors f_n may be estimated by the use of an equivalent truncated right circular cone in place of the actual rectangular cross-section cone. The view factor directly to space, f_o, from the center of the patch can be calculated from a formula given by Jakob (op. cit., p. 12)

$$F_{ps} = f_o = \frac{2}{\pi} \left[\frac{b}{\sqrt{b^2 + \ell^2}} \arcsin \left(\frac{a}{\sqrt{a^2 + b^2 + \ell^2}} \right) + \frac{a}{\sqrt{a^2 + \ell^2}} \arcsin \left(\frac{b}{\sqrt{a^2 + b^2 + \ell^2}} \right) \right] \qquad \text{(II-20)}$$

The cone length is ℓ (Figure I-1 in Appendix I) and the cone mouth area 2a x 2b. In the single-stage cooler, a = 1.375 inches, b = 1.75 inches, and ℓ = 4.6 inches. From equation (II-20), f_o is then equal to 0.126. In a right circular cooler

$$f_o = \sin^2 \psi_o$$

where ψ_o is the angle between the patch normal and ray 0, as shown in the attached sketch. The cone height (distance from patch center to diameter AD along normal to patch) is 4. 6 inches in the single-stage cooler, so that the equivalent cooler has a radius $r = 4.6 \tan \psi_o = 1.74$ inches at the opening AD. The angles ψ_n (Figure II-1) can be determined by means of the law of sines and law of cosines, which yield the equation

$$\sin \psi_n = \frac{r_2 \sin [(2n+1)\theta]}{[r_2^2 + r_1^2 \cos^2 \theta - 2 r_1 \cos \theta r_2 \cos (<2n+1>\theta)]^{1/2}} \quad \text{(II-21)}$$

where

θ = cone half angle

r_1 = distance along cone surface from apex to patch

r_2 = distance along cone surface from apex to mouth

The geometry of the cooler is shown in Figure I-1 in Appendix I. For a given maximum patch look angle ϕ (equation I-4 in Appendix I), this may be written

$$\sin \psi_n = \frac{\sin [(2n+1)\theta]}{[1 + \sin^2(\phi - \theta) \cos^2 \theta + \sin(\phi - \theta) \cos \theta \cos (<2n+1>\theta)]^{1/2}} \quad \text{(II-22)}$$

As in Appendix I, we will assume that the patch area is concentrated at its center, i. e., that the view factors from the center of the patch equal the values averaged over the patch area. For a truncated right circular cone in which rays at angles between ψ_{n-1} and ψ_n to the patch normal require n reflections to reach cold space, the view factor is given by

$$f_n = 1/\pi \int_{\varphi=0}^{2\pi} \int_{\vartheta=\psi_{n-1}}^{\psi_n} \sin \vartheta \cos \vartheta \, d\vartheta \, d\varphi = \sin^2 \psi_n - \sin^2 \psi_{n-1} \quad \text{(II-23)}$$

The integral is written in terms of spherical coordinates at the patch center with the pole along the patch normal; ϑ is the pole angle and φ the azimuthal angle. From equations (II-13), (II-22), and (II-23) we see that the effective patch-to-cone emissivity for a given maximum look angle depends only on the emissivity of the gold-coated surfaces and the half angle of the cone.

For the equivalent right circular single-stage cooler, equation (II-21) yields

ψ_1 = $55^o\ 23'$

ψ_2 = $82^o\ 45'$

Rays from the patch center at angles to the normal between ψ_o and ψ_1 require one reflection to go to space, those between ψ_1 and ψ_2, two reflections, and those between ψ_2 and 90 degrees, three reflections. The f_n for the right circular equivalent of the single-stage cooler are then

$$
\begin{aligned}
f_0 &= \sin^2 \psi_0 = 0.126 \\
f_1 &= \sin^2 \psi_1 - \sin^2 \psi_0 = 0.570 \\
f_2 &= \sin^2 \psi_2 - \sin^2 \psi_1 = 0.291 \\
f_3 &= 1 - \sin^2 \psi_2 = 0.013
\end{aligned}
\tag{II-24}
$$

and

$$\sum_{n=0} n\, f_n = 1.19 \tag{II-25}$$

From equations (II-17) and (II-19) the average surface emissivity of the 200 degrees K cooler is

$$\epsilon_g = 0.086 \tag{II-26}$$

The back area of the patch assembly and cylinder surface contain holes, connecting wires, and surface cavities. The surface emissivity of the cone walls is therefore lower than the average. For this reason, use of the above value to estimate the performance of the two-stage cooler, in which the patch radiates to space from both sides and the cylinder is eliminated, yields upper limits for the patch temperature (i. e., minimum performance values).

Now the radiation exchange is a minimum for specular reflection at the enclosing surface for close-spaced, coaxial, or concentric geometry (M. Jakob, "Heat Transfer", Vol. II, Wiley and Chapman and Hall, 1957, p. 49). Such a geometry is assumed in equation (II-17) and thus in the result (II-26). We may therefore conclude that the average gold surface emissivity in the single-stage cooler is less than 0.086. A lower limit is obtained by assuming the cylinder to be effectively black. The radiative transfer coefficient then becomes

$$H_i = \sigma \epsilon_g \left[A_r \sum_{n=0} n\, f_n + A_b \right] \tag{II-27}$$

And the average gold surface emissivity (cone and back of patch) is reduced to $\epsilon_g = 0.062$.

APPENDIX III

EFFECTIVE PATCH TO CONE EMISSIVITIES IN TWO-STAGE COOLER

The general technique of calculating the effective patch to cone emissivity for truncated right circular cones using the method of multiple reflections is described in Appendix II. From equation (II-13) in that Appendix, the effective patch to cone emissivity of the second stage for a truncated right circular cone is

$$\epsilon_{pc}^{(2)} = 1 - \sum_{n=0} f_n (1 - \epsilon_g)^n = 1 - \rho_{pc}^{(2)} \qquad \text{(III-1)}$$

where

$\rho_{pc}^{(2)}$ = effective reflectivity of cone for patch radiation

ϵ_g = true surface emissivity of gold-coated cone

$1 - \epsilon_g$ = true surface reflectivity of cone

f_n = view factor from patch to cone wall area initially intercepted by rays from the patch that require n cone reflections to go to cold space

The view factor f_n is also equal to the fraction of emitted patch radiation that requires n reflections in a perfectly reflecting cone to go to cold space.

The angle ψ_n can be determined by means of the multiple reflections shown in Figures III-1 and III-2. Thus rays from the patch center at polar angles between ψ_1 and ψ_2 require two reflections at the cone to go out the cone mouth. Stated another way, an observer at the patch center looking out at angles to the patch normal between ψ_1 and ψ_2 sees cold space by two reflections in the cone walls. The angle ψ_n in a truncated right circular cone is given by equation (II-21) in Appendix II, which is

$$\sin \psi_n = \frac{r_2 \sin \left[(2n + 1)\, \theta \right]}{\left[r_2^2 + r_1^2 \cos^2 \theta - 2 r_1 \cos \theta \, r_2 \cos (\langle 2n + 1 \rangle \, \theta) \right]^{1/2}} \qquad \text{(III-3)}$$

where

θ = half-angle of cone

r_1 = distance from cone apex along cone surface to patch

r_2 = distance from cone apex along cone surface to mouth

Full Scale

$\theta = 9^{o}$
$r_1 = 1.247''$
$r_2 = 4.59''$
$\ell = 3.3''$

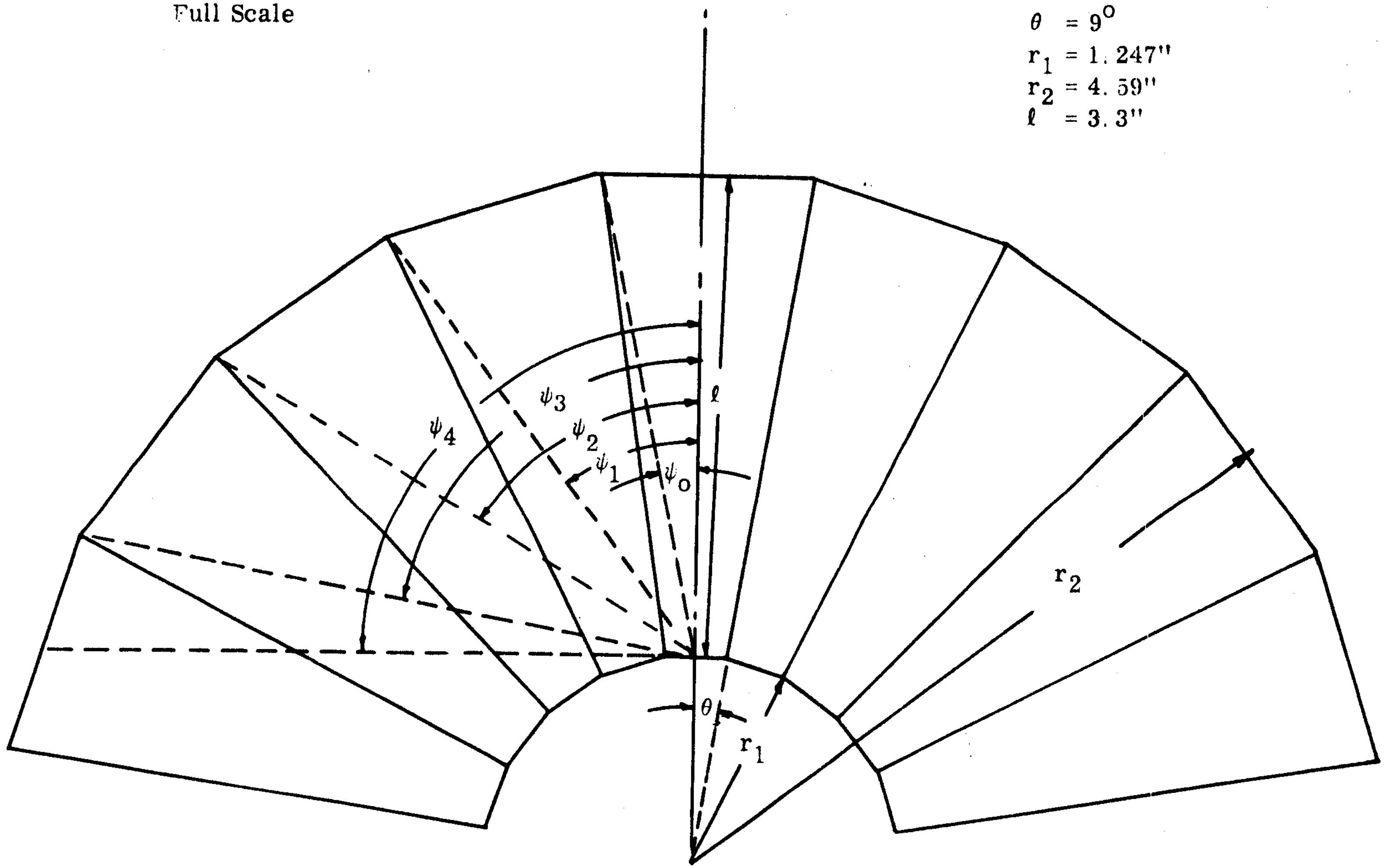

Figure III-1 Vertical Plane of Second Stage Reflected in Cone Walls

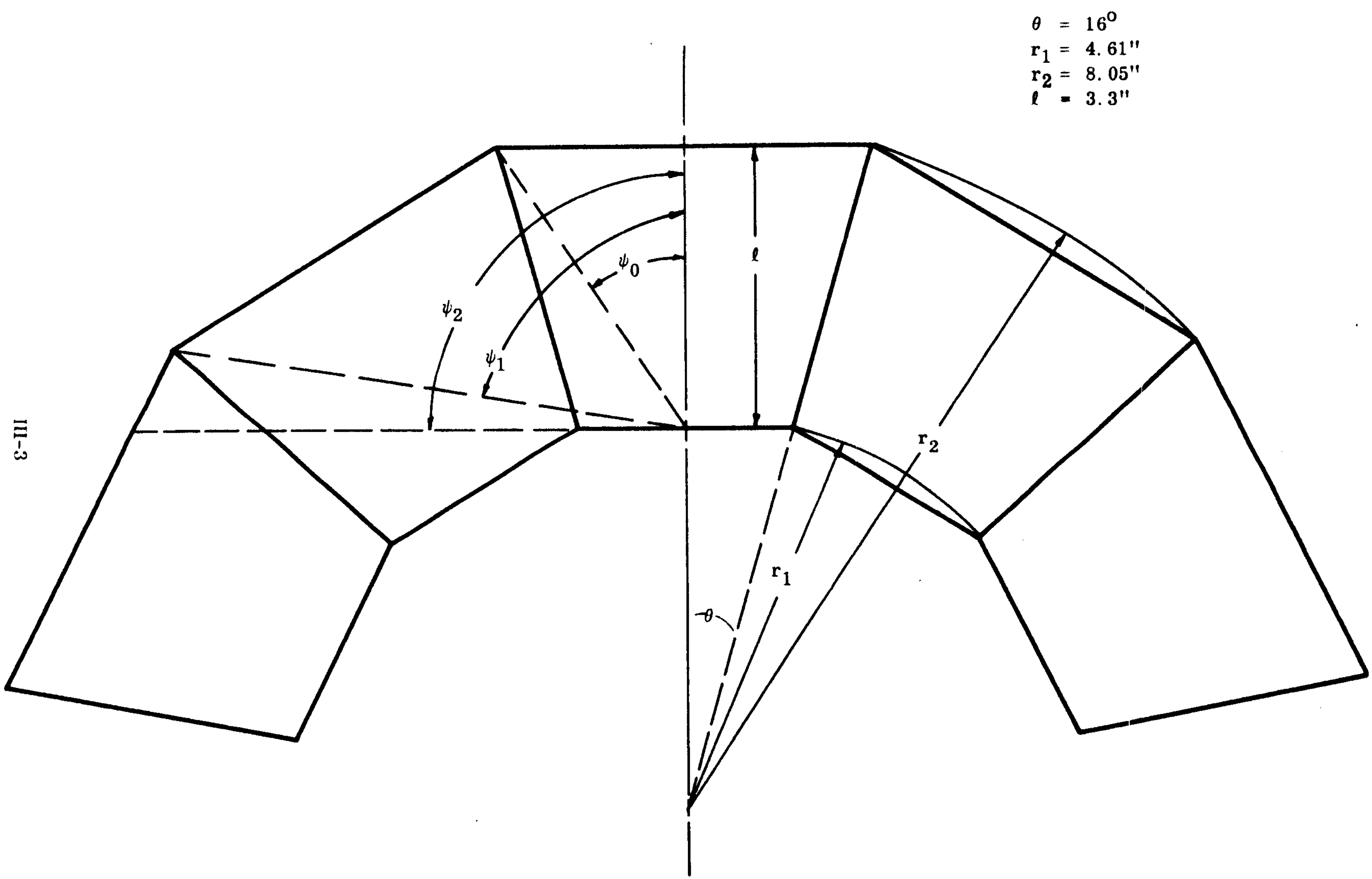

Figure III-2 Horizontal Plane of Second Stage Reflected In Cone Walls 2/3 Scale

Figures III-1 and III-2 show the multiple reflections of truncated right circular cones having the geometry of the vertical plane and horizontal plane, respectively of the second stage of the two-stage cooler. These geometries have been used in place of the equivalent right circular cone employed in Appendix I because of the large aspect ratio of the patches in the two-stage cooler.

The values of sin ψ_n were calculated for the vertical and horizontal geometries using equation (III-3). The view factors f_n were then determined from equation (III-2). The results are given in Tables III-1 and III-2. Note that $f_o = \sin^2 \psi_o$.

Table III-1

View Factors for Vertical Geometry of Second Stage

n	$\sin \psi_n$	$f_n = \sin^2 \psi_n - \sin^2 \psi_n - 1$
0	0.212	0.045
1	0.589	0.302
2	0.849	0.373
3	0.978	0.236
4	1	0.044

Table III-2

View Factors for Horizontal Geometry of Second Stage

n	$\sin \psi_n$	$f_n = \sin^2 \psi_n - \sin^2 \psi_{n-1}$
0	0.556	0.309
1	0.968	0.662
2	1	0.029

The sum of all view factors Σf_n is unity for a given geometry according to the definition of view factor (M. Jakob, "Heat Transfer", Vol. II, Wiley and Chapman and Hall, 1957, p. 10).

The values of partial reflectivities, $f_n (1 - \epsilon_g)^n$, are listed in Table III-3 for ϵ_g equal to 0.02 and 0.086.

Table III-3

Partial Patch to Cone Reflectivities for Second Stage

n	Vertical ϵ_g 0.02	Vertical 0.086	Horizontal 0.02	Horizontal 0.086
0	0.045	0.045	0.309	0.309
1	0.296	0.276	0.605	0.649
2	0.358	0.312	0.024	0.028
3	0.222	0.180		
4	0.041	0.031		

By adding the partial reflectivities and subtracting the sum from unity, we obtain the effective patch to cone emissivity according to equation (III-1). The results are given in Table III-4.

Table III-4

Effective Patch to Cone Emissivity for Second Stage

ϵ_g	Vertical	Horizontal	Average
0.02	0.038	0.014	0.026
0.086	0.156	0.062	0.109

The effective patch to cone emissivity for the first stage was also determined using the above procedure. The results are shown in Table III-5 for gold surface emissivities of 0.02 and 0.086.

Table III-5

Effective Patch to Cone Emissivity for First Stage

ϵ_g	Vertical	Horizontal	Average
0.02	0.024	0.010	0.017
0.086	0.103	0.044	0.0735

APPENDIX IV

EXTERNAL THERMAL LOAD ON FIRST STAGE (SECOND-STAGE CONE)

The radiative coupling between the modified second-stage cone and the external sources may be estimated by means of the vertical-and horizontal-plane ray traces described in Section 2.2. As seen by the second-stage cone, the external sources may be considered to lie in the plane of the mouth of the first-stage cone and to consist of four areas, designed by A_1 and A_2 in Figure IV-1. Each area extends 3.8 inches* from the mouth edge and is as wide as the mouth of the second-stage cone.

The radiant emittance (watts/cm^2 emitted) of the second-stage cone is much less than that of the external sources, so that the thermal load on the first stage from an external source is very nearly

$$\Phi_{x-c2} = A_{c2} F_{c2-x} W_x \alpha_x \qquad \text{(IV-1)}$$

where

A_{c2} = outside area of the second-stage cone

F_{c2-x} = view factor from the second-stage cone to the external source (as seen by reflection)

W_x = radiant emittance of the external source

α_x = absorption factor for external radiation incident on the second-stage cone

However, we have for diffuse emitters (M. Jakob, "Heat Transfer", Volume II, Wiley, 1957, p. 9)

$$A_{C2} F_{C2-x} = 2A_1 F_{A1-C2} + 2A_2 F_{A2-C2} \qquad \text{(IV-2)}$$

where

F_{A1-C2} = view factor from A_1 to the second-stage cone

F_{A2-C2} = view factor from A_2 to the second-stage cone

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*Estimated by means of a scale drawing

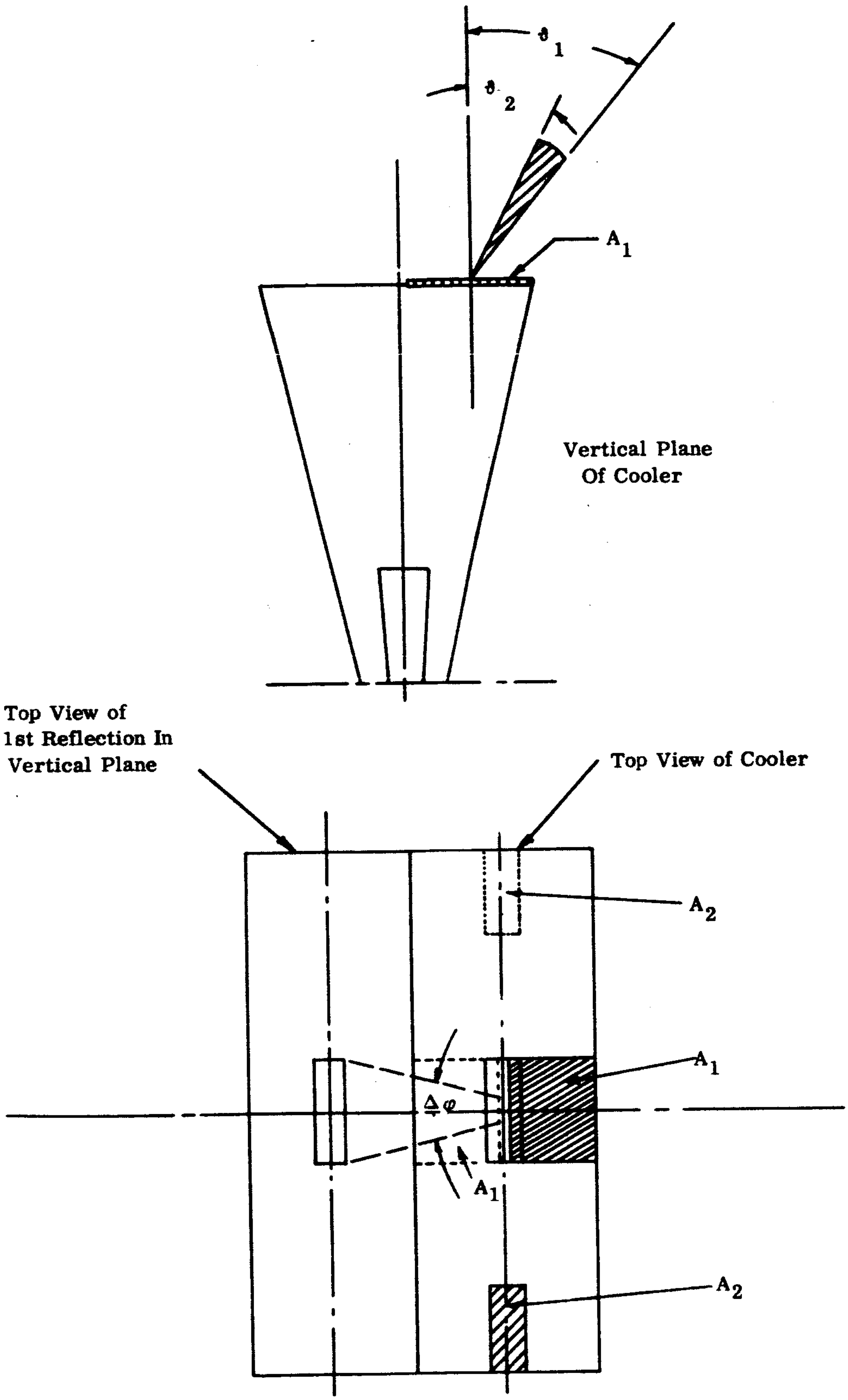


Figure IV-1 Radiative Load on 1st Stage (2nd Stage Cone) From External Sources

The average view factors may be estimated from the following equation (See Jakob, op, cit., eq. (31-17)

$$F_{Ai-C2} = \frac{1}{2\pi} \int_{\varphi_1}^{\varphi_1+\Delta\varphi} \int_{\vartheta_1}^{\vartheta_2} \cos\vartheta \sin\vartheta \, d\vartheta \, d\varphi \qquad \text{(IV-3)}$$

The factor 1/2 assumes the average value of $\int \cos\vartheta \sin\vartheta \, d\vartheta$ is one-half the maximum. The maximum value is at the edge of the cone mouth and is determined by the maximum ϑ range, ϑ_1 to ϑ_2. The quantity $\sin\vartheta \, d\vartheta \, d\varphi$ is an element of solid angle in spherical coordinates ($\frac{1}{r^2} \cos\phi_2 \, dA_2$ in Jakob's notation) centered on A_i, as shown in Figure IV-1. The factor $\cos\vartheta$ ($\cos\phi_1$ in Jakob's notation) accounts for the diffuse nature of the radiation and $\frac{1}{\pi}$ normalizes the view factor to a maximum value of unity. The view factor from one surface to another is unity when all rays emitted by the first surface strike the second surface. Integrating (IV-3).

$$F_{Ai-C2} = \frac{\Delta\varphi}{4\pi} \left[\sin^2\vartheta_2 - \sin^2\vartheta_1\right] \qquad \text{(IV-4)}$$

In the vertical plane, ϑ_2 = 43 degrees and ϑ_1 = 31.5 degrees; in the horizontal plane, ϑ_2 = 65 degrees and ϑ_1 = 60 degrees. The angular range $\Delta\varphi$ is set by the first reflection of the second-stage cone in the first-stage cone. As seen along the patch normal, it is the angle subtended at the center of A_i by the side of the second-stage cone once reflected in the first-stage cone. Equation (IV-3) assumes that the limiting ϑ values are the same throughout $\Delta\varphi$. In addition, the absorption at the second-stage cone is less for rays out of the horizontal and vertical planes. To account for these effects, we will assume that the areas at the mouth of the first-stage cone are limited in their widths to the dimensions at the mouth of the second-stage cone (see Figure IV-1) and that the absorption factors at the second-stage cone are constant in $\Delta\varphi$ and equal to the average value in the ϑ range. Note that the top view of the first reflection in the vertical plane shown in Figure IV-1 is shortened in the vertical direction by cos (2 x 13.5 degrees) = 0.891.

In the vertical plane, $\Delta\varphi$ = 28 degrees = 0.49 radian. This value was obtained using the dimensions and angles of the first-stage design and the dimensions of the modified (final) second-stage cone, (Section 2.1). Thus,

$$F_{A1-C2} = 7.5 \times 10^{-3}$$

And in the horizontal plane, $\Delta\varphi$ = 3 degrees = 5.2×10^{-2} radian, and

$$F_{A2-C2} = 3 \times 10^{-4}$$

Radiative coupling between the second-stage and external sources is therefore much weaker in the horizontal direction than in the vertical direction.

We will assume that the second-stage cone sees the two A_2 (horizontal direction) areas as blackbodies at the maximum radiometer base temperature of 35 degrees C. The emittance of these external sources is therefore $W_b = 5.1 \times 10^{-2}$ watts/cm^2. The A_1 (vertical direction) area nearest the spacecraft is seen at the same temperature, while the other A_1 passes infrared emission and reflected sunlight from the earth. The sunlight reflected from the earth (earthshine) will be expressed as an average equivalent source emittance for a sun-synchronous, polar orbiting earth-oriented spacecraft. Because the thermal time constant of the first stage is very long, at least the order of the time it takes the spacecraft to orbit the earth (Appendix V), only the earthshine value averaged over a spacecraft period is needed.

The average earth solar reflection factor, A, is assumed to be independent of the angle of incidence at the earth and the angle of view from the cooler to the earth's surface. In addition, it is assumed that the amount of reflected sunlight is directly proportional to the cosine of the incidence angle at the earth's surface (i. e., to the illuminated area projected in the sun's direction). For a satellite in sun-synchronous polar orbit, as shown in Figure IV-2, the cosine of the earth incidence angle is given by

$$\cos \theta' = \cos \theta \cdot \cos \phi \qquad \text{(IV-5)}$$

where

θ = earth latitude
ϕ = great circle angle from spacecraft subpoint to point on earth's surface seen by second-stage cone

In the vertical plane, the look angle which intercepts the earth (from 600 nautical miles altitude) varies from 31.5 degrees to 43 degrees, so that ϕ varies from 31.5 degrees to 12 degrees (see Figure IV-2). The latitude θ covers the range from 0 degree to 90 degrees. Since the cooler sees an earth which is sun illuminated half the time, the average equivalent earthshine emittance is

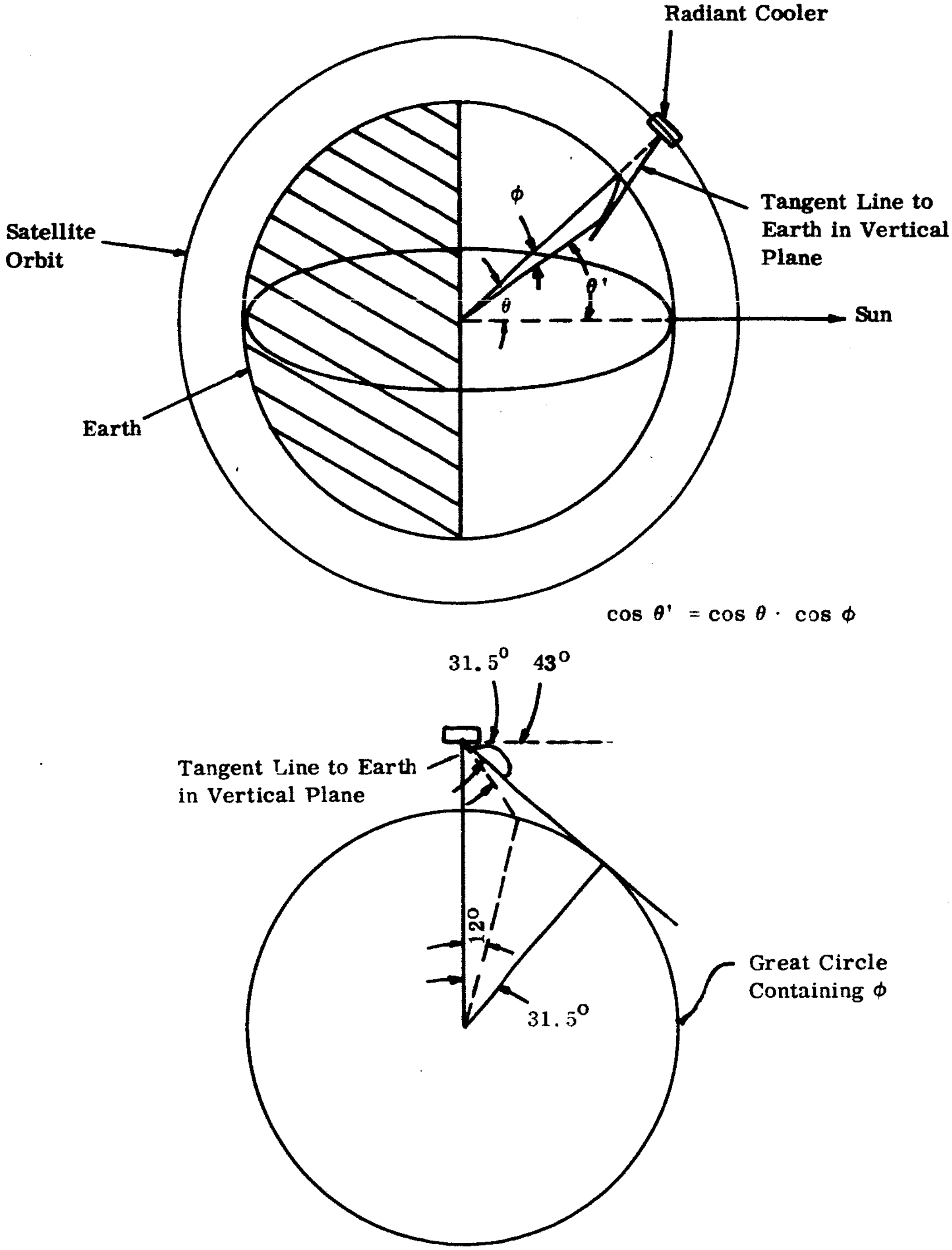
$$W_s = (1/2) s_o A \times \frac{1}{0.34} \int_{12^\circ}^{31.5^\circ} \cos \phi \cdot d\phi \times \frac{2}{\pi} \int_0^{\pi/2} \cos \theta \cdot d\theta \qquad \text{(IV-6)}$$

where

s_o = solar constant = 0.14 watts/cm^2
A = average solar reflection factor for earth = 0.4

The factor 0.34 is the angular range from 12 degrees to 31.5 degrees in radians. Integrating (IV-6)

$$W_s = 1.65 \times 10^{-2} \text{ watts/cm}^2 \qquad \text{(IV-7)}$$



$$W_s = S_o A \times 1/2 \times 1/0.34 \int_{12^o}^{31.5^o} \cos\phi \cdot d\phi \times 2/\pi \int_0^{\pi/2} \cos\theta \cdot d\theta$$

$$= 0.295\ S_o A = 1.65 \times 10^{-2}\ \text{watts/cm}^2$$

Figure IV-2 Calculation of Average Earthshine Emittance

The solar radiation not reflected by the earth is absorbed by it and radiated to space at infrared wavelengths. The average infrared emittance of the earth, W_e, is therefore determined by

$$4\pi R'^2 W_e = \pi R'^2 s_o (1 - A)$$
$$W_e = (1/4) s_o (1 - A) = 2.1 \times 10^{-2} \text{ watts/cm}^2 \qquad \text{(IV-8)}$$

where R' is the earth's radius. If the earth were a uniform blackbody in the infrared as seen from space (which it is not), its temperature would be 247 degrees K.

Next, we need to know the second-stage cone absorption factors for earthshine and infrared radiation. The solar absorption factor, α_g, for a single reflection from a gold surface was calculated using the spectral reflection of gold given by C. W. Allen ("Astrophysical Quantities", U. of Lon., The Athlone Press, 1963, p. 104) and the spectral irradiance of the sun's rays outside the earth's atmosphere given in the "Handbook of Geophysics" (USAF, Macmillan, 1960, Chapt. 16). The result was $\alpha_g \cong 0.22$ or a solar reflectance of 0.78. We will assume that the infrared emissivity (absorptivity) of the gold surface equals the average value of the gold surfaces in the single-stage, 200 degrees K radiant cooler (0.086). If the radiation is reflected between the first-stage cone and the second-stage cone with five reflections at the second-stage cone, the absorption factor is

$$\alpha_{\max_1 v} = \alpha R \, [1 + R^2 + R^4 + R^6 + R^8] = \frac{R \, (1 - R^{10})}{1 + R} \qquad \text{(IV-9)}$$

where $R = 1 - \alpha_g$ is the reflectivity. In the vertical plane, the maximum number of second-stage cone reflections is five, so the maximum absorption factor is 0.40 for earthshine and 0.362 for infrared radiation. The minimum absorption factors ($\alpha_g R$) are 0.172 and 0.079, respectively. Since the external radiation at 31.5 degrees passing through A_1 undergoes from 5 to 1 reflections at the second-stage cone, the average absorption factors at this angle are 0.287 for earthshine and 0.181 for infrared. Rays at 43 degrees through A_1 undergo only one reflection at the second-stage cone. The average absorption factors in the vertical plane for all rays through A_1 are then $\alpha_{s,v} = 0.23$ for earthshine and $\alpha_{ir,v} = 0.13$ for infrared. In the horizontal plane, the average infrared absorption factor is $\alpha_{ir,h} = 0.095$.

Using the facts that $A_1 = 3.8 \times 4.5 \text{ in}^2 = 110 \text{ cm}^2$ and $A_2 = 3.8 \times 1.5 \text{ in}^2 = 36.7 \text{ cm}^2$, we are finally in a position to calculate the thermal load on the first stage (second-stage cone) produced by external sources at angles beyond the maximum look angles of the first-stage patch.

The result is

$$\Sigma\Phi_{x\text{-}C2} = 2\,\{A_1\, F_{A1\text{-}C2}\, [\alpha_{ir,v}\,(W_b + W_e) + \alpha_{s,v} W_s] +$$

$$2A_2\, F_{A2\text{-}C2}\, \alpha_{ir,h}\, W_b\}$$

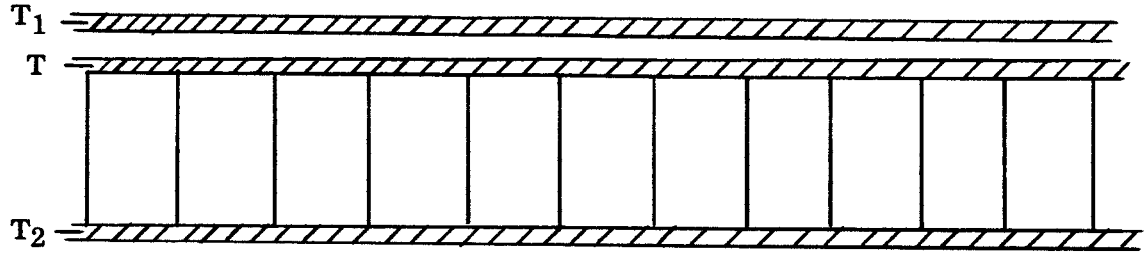
$$\Sigma\Phi_{x\text{-}C2} = 22 \text{ milliwatts} \qquad \text{(IV-10)}$$

The factor 2 in front accounts for the double-ended cone structure. The first stage radiates about 300 milliwatts at a temperature of 105 degrees K. The above load increases this by 7.3 percent and therefore increases the patch temperature by about $\frac{7.3}{4}$ percent or 1.9 degrees K.

APPENDIX V

THERMAL TIME CONSTANT OF OUTER CONE FACING

Consider a cone facing at a temperature T located between a radiant shield at T_1 and an inner cone facing at T_2, as shown in Figure V-1. The facing may be considered the outer facing of the first-stage cone.



$$T_1 > T > T_2$$

Figure V-1 Cone Facing and Surroundings

In the steady-state, the temperature T is determined by the following thermal balance equation and designated T_o

$$\frac{\epsilon_g}{2} \sigma A \left(T_1^{\,4} - T_o^{\,4}\right) = \frac{\epsilon_g}{2} \sigma A \left(T_o^{\,4} - T_2^{\,4}\right)$$

where

ϵ_g = emissivity of surfaces of facings and radiant shield

A = surface area of one side of facing

σ = Stefan-Boltzmann constant

This equation assumes that ϵ_g is much less than two and that there is plane-parallel geometry. Thermal conductivity through the supports has been neglected. The equation may be written as

$$\epsilon_g \sigma A\, T_o^{\,4} = \frac{\epsilon_g}{2} \sigma A \left(T_1^{\,4} + T_2^{\,4}\right) = W_o$$

where W_o is the radiant power absorbed by the cone facing.

If there is a small change, ΔW, in absorbed power, the temperature of the facing will begin to change. The change is not instantaneous, however, because of the thermal capacity of the facing. The thermal balance equation during the change in facing temperature becomes

$$C \frac{\partial T}{\partial t} + \epsilon_g \sigma A T^4 = W_o + \Delta W$$

where C is the thermal capacity of the facing and t is time. The change in incident power therefore produces a change in the amount of thermal energy stored in the facing as well as in its temperature. Since a small change in W will produce a small change, ΔT, in temperature we may write $T = T_o + \Delta T$. We then have

$$C \frac{\partial \Delta T}{\partial t} + \epsilon_g \sigma A T_o^4 \left(1 + \frac{\Delta T}{T_o}\right)^4 = W_o + \Delta W$$

For $\Delta T \ll T_o$, this becomes

$$C \frac{\partial \Delta T}{\partial t} + 4 \epsilon_g \sigma A T_o^3 \Delta T + \epsilon_g \sigma A T_o^4 = \Delta W + W_o$$

or

$$C \frac{\partial \Delta T}{\partial t} + 4 \epsilon_g \sigma A T_o^3 \Delta T = \Delta W$$

The solution to the above differential equation is

$$\Delta T = \frac{\Delta W}{4 \epsilon_g \sigma A T_o^3} \left(1 - \epsilon^{-t/\tau}\right)$$

$$\tau = \frac{C}{4 \epsilon_g \sigma A T_o^3}$$

Thus the change in temperature ΔT approaches a steady-state value with a time constant τ. The factor $4 \epsilon_g \sigma A T_o^3$ is the radiative thermal conductance. Writing

$$C = \rho c \delta A$$

where

ρ = density of facing material
c = specific heat of facing material
δ = thickness of facing

the time constant becomes

$$\tau = \frac{\rho c \delta}{4 \epsilon_g \sigma T_o^3}$$

For an aluminum facing at -50 degrees C = 223 degrees K,

$$c = .8\ 0 \text{ joules/gm degrees C}$$
$$\rho = 2.7 \text{ gms/cm}^3$$

For $\delta = 2.5 \times 10^{-2}$ inch $= 6.35 \times 10^{-2}$ cm and $\epsilon_g = 0.1$, $\tau = 5.45 \times 10^3$ seconds = 1.51 hours.

The thermal time constant of the cone facing is therefore the order of the time it takes the spacecraft to orbit the earth. The thermal lag of the cone facing and other structures smooths out the variations in power absorbed at the outer surface of the radiant cooler. Absorbed power averaged over a spacecraft orbit can therefore be used to determine the effect of outer surface thermal load on radiant cooler operation.

APPENDIX VI

VIEW FACTOR FROM SECOND-STAGE PATCH TO FIRST-STAGE CONE

The second-stage patch must see none or only a very little of the first-stage cone in order to attain the desired temperature range. That is, it is to be thermally coupled to the first-stage patch (and the second-stage cone mounted on it) and to cold space but not to sources of higher temperature radiation, such as the first-stage cone. As long as the edges of the second-stage patch do not see the first-stage cone directly (i.e., without reflection in the second-stage cone), the view factor from the center of the patch can be used as an estimate for the entire patch area. In addition, if the patch does not directly view the first-stage cone at its edges, it will not view the cone directly at any position on the patch.

When the maximum angle to the cone axis set by the nth reflection of the second-stage cone in itself is less than or equal to the maximum angle set by the nth reflection of the first-stage cone in the second-stage cone, no second-stage patch radiation will reach the first-stage cone after n reflections in the second-stage cone. This is illustrated by Figure VI-1, which shows the horizontal plane of half of each stage of the cooler. The first and second reflections of the (modified) second stage and the first reflection of the first stage, both in the second-stage cone, are also shown. The angle η, from the cone axis to the first reflection of the first-stage cone is equal to the angle ϕ, from the cone axis to the first reflection of the second-stage cone. As a result, radiation from the center of the patch does not view the first-stage cone after one reflection in the second-stage cone.

The angle ϕ_n, subtended at the patch center by the nth reflection of the second-stage cone in itself is determined in Appendix III. The angle η_n subtended at the patch center by the nth reflection of the first-stage cone in the second-stage cone can be calculated from the following equations, derived using the laws of sines and cosines (see Figure VI-1).

$$c_n = 2\,(\ell^{(1)} + r_1 \cos\theta)\,\sin\theta \qquad \text{(VI-1)}$$

$$d_n^2 = b^2 + c_n^2 + 2bc_n \cos\theta \qquad \text{(VI-2)}$$

$$\sin\mu_n = \frac{b}{d_n}\sin\theta \qquad \text{(VI-3)}$$

$$e_n^2 = d_n^2 + (\ell^{(1)})^2 - 2d_n\,\ell^{(1)}\sin(\theta + \mu_n) \qquad \text{(VI-4)}$$

$$\sin\eta_n = \frac{d_n}{e_n}\cos(\theta + \mu_n) \qquad \text{(VI-5)}$$

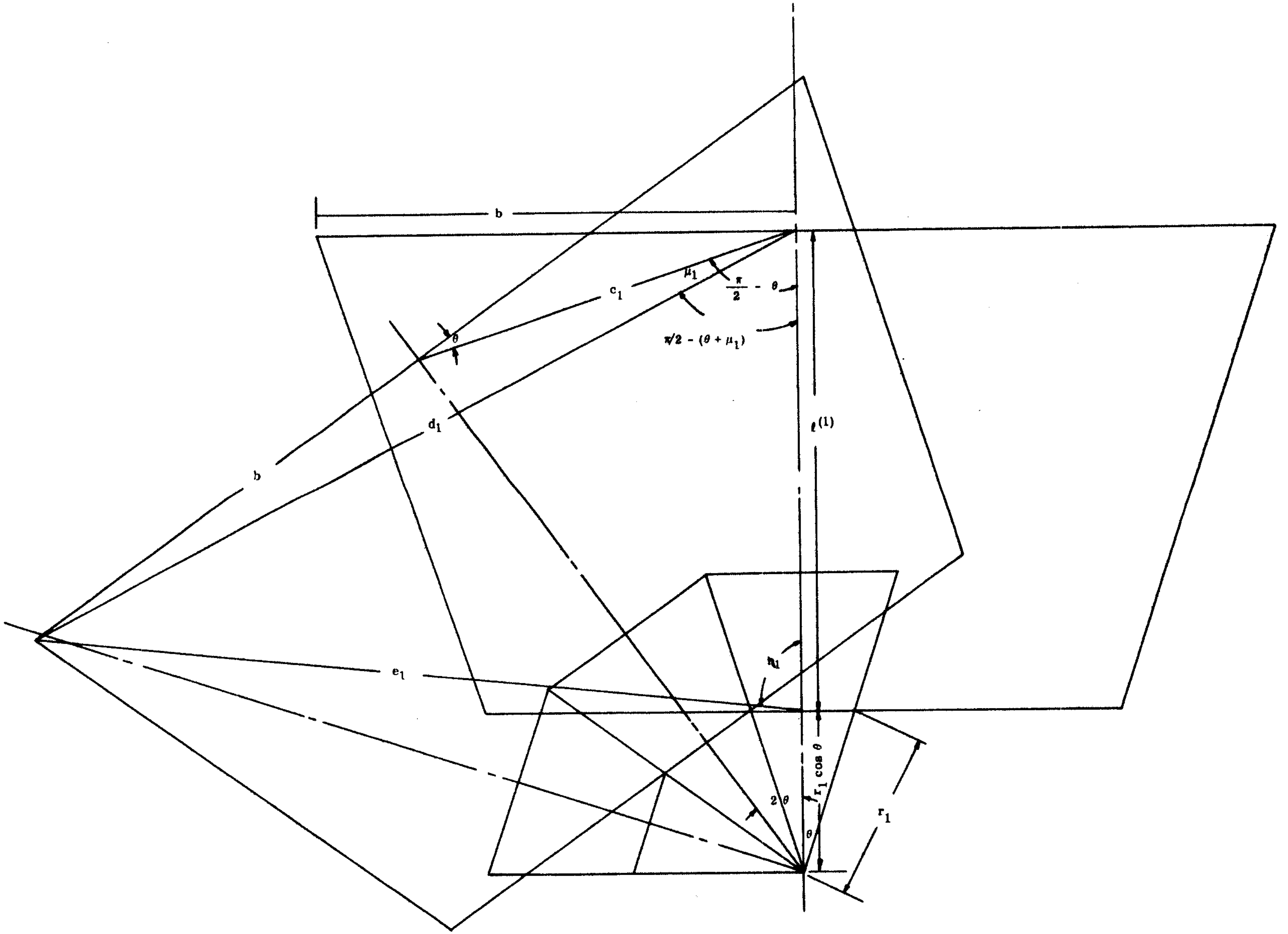


Figure VI-1 Second-Stage Patch to First-Stage Cone View Factor, Horizontal Plane where

$\ell^{(1)}$	=	cone length of first stage
r_1	=	distance along cone from apex to patch in second stage
θ	=	half angle of second-stage cone
b	=	half width of first-stage cone mouth

Knowing ϕ_n and η_n, the view factor from the center of the second-stage patch to the nth second-stage reflection of the first-stage cone is, in a truncated right circular cone

$$F_{p2-c1}^{(n)} = \sin^2 \phi_n - \sin^2 \eta_n \tag{VI-6}$$

If ϕ_n is less than or equal to η_n, the view factor is zero. The complete view factor is then

$$F_{p2-c1} = \sum_n F_{p2-c1}^{(n)} \tag{VI-7}$$

The values of the view factors from the center of the second-stage patch to the first-stage cone were calculated for truncated right circular cones having the geometry of the vertical and horizontal planes of the cooler. The results are listed in Table VI-1 for the initial second-stage design (Section 3.3) and in Table VI-2 for the modified design (Section 3.4).

Table VI-1

View Factors from Center of Patch to First-Stage Cone in Initial Second Stage

	vertical			horizontal		
n	ϕ_n	η_n	$F_{p2-c1}^{(n)}$	ϕ_n	η_n	$F_{p2-c1}^{(n)}$
1	$41^\circ 45'$	$42^\circ 50'$	0	$84^\circ 34'$	$86^\circ 3'$	0
2	$67^\circ 16'$	$65^\circ 40'$	0.0204	90°	90°	0
3	90°	$86^\circ 10'$	0.0045		F_{p2-c1} =	0
	F_{p2-c1} =		0.0249			

Table VI-2
View Factors from Center of Patch to First-Stage Cone in Modified Second Stage

	vertical			horizontal		
n	ϕ_n	η_n	$F^{(n)}_{p2-c1}$	ϕ_n	η_n	$F^{(n)}_{p2-c1}$
1	$36^\circ 5'$	$38^\circ 50'$	0	$80^\circ 24'$	$80^\circ 24'$	0
2	$58^\circ 6'$	$57^\circ 53'$	0.0034	90°	90°	0
3	$77^\circ 58'$	$76^\circ 10'$	0.0137		F_{p2-c1} =	0
4	90°	$84^\circ 16'$	0.0100			
		F_{p2-c1} =	0.0271			

Values are not given for n = 0 (no reflections) since the stage was designed so the first-stage cone cannot be seen directly from any point on the patch. In addition, because patch radiation leaves at a maximum angle of 90 degrees to the cone axis, 90 degrees is the effective limit for both ϕ_n and η_n.

To estimate the effect of the small view factor on the temperature of the second stage, consider the radiative coupling between the second-stage patch and first-stage cone. It is given by

$$\Phi_{p2-c1} = A_{p2} F_{p2-c1} \epsilon_g \sigma (T_{c1}^4 - T_{p2}^4) \qquad \text{(VI-8)}$$

where

A_{p2} = radiating area of the second stage

ϵ_g = emissivity of gold surface on first-stage cone

σ = Stefan-Boltzmann constant

T_{c1} = temperature of first-stage cone

T_{p2} = temperature of second-stage patch

Radiation that reaches the first-stage cone is reflected only once at that surface, so that the absorptivity equals the emissivity of the cone surface. Using the fact that T_{c1}^4 is much larger than T_{p2}^4 and expressing the coupling power as a fraction of the total patch power, we obtain

$$\frac{\Phi_{p2-c1}}{\Phi_{p2}} = \epsilon_g \; F_{p2-c1} \left(\frac{T_{c1}}{T_{p2}}\right)^4 \qquad \text{(VI-9)}$$

For $\epsilon_g = 0.086$ (Appendix II), $T_{c1} = 199$ degrees K with the earth shield in place (Section 4.3). Using the average view factor for the vertical and horizontal planes (0.01355) and $T_{p2} = 77$ degrees K,

$$\frac{\Phi_{p2-c1}}{\Phi_{p2}} = 7.8 \times 10^{-2} \qquad \text{(VI-10)}$$

This would increase the patch temperature by about 2 percent or 1.5 degrees K. For $\epsilon_g = 0.02$, $T_{c1} = 248$ degrees K (Section 4.2), and for $T_{p2} = 67$ degrees K, the temperature increase is 0.6 degree K.

APPENDIX VII

VIEW FACTOR FROM OUTSIDE VERTICAL COOLER SURFACE TO EARTH

The view factor from an outside vertical radiant cooler surface to the earth equals the fraction of diffuse radiation emitted by the surface which strikes the earth. In spherical coordinates at the vertical cooler surface, with the pole along a surface normal, the view factor is (See M. Jakob, "Heat Transfer", Volume II, Sections 31.5 and 31.6 for a discussion of view factors)

$$F_{se} = \frac{1}{\pi} \int_{-\varphi_o}^{\varphi_o} \int_{\vartheta(\varphi)}^{\pi/2} \sin\vartheta \cos\vartheta \, d\vartheta \, d\varphi \qquad \text{(VII-1)}$$

The geometry and coordinate system are shown in Figure VII-1. The angle φ_o is the angle from the spacecraft nadir to a tangent line to the earth's surface, 58.5 degrees for a spacecraft altitude of 600 nautical miles. The lower limit to the polar angle, ϑ, is a function of the azimuthal angle,. φ, going from $\pi/2 - \varphi_o$ at $\varphi = 0$ to $\pi/2$ at $\varphi = \varphi_o$. Integrating with respect to ϑ, we obtain

$$F_{se} = \frac{1}{2\pi} \int_{-\varphi_o}^{\varphi_o} [1 - \sin^2 \vartheta(\varphi)] \, d\varphi = \frac{1}{2\pi} \int_{-\varphi_o}^{\varphi_o} \cos^2 \vartheta(\varphi) \, d\varphi \qquad \text{(VII-2)}$$

But, from Figure VII-1,

$$\cos^2 \vartheta(\varphi) = \frac{\tan^2 \varphi_o - \tan^2 \varphi}{\tan^2 \varphi_o - \tan^2 \varphi + \sec^2 \varphi} \qquad \text{(VII-3)}$$

Using the relation $\sec^2 X - \tan^2 X = 1$, this becomes

$$\cos^2 \vartheta(\varphi) = 1 - \frac{\sec^2 \varphi}{\sec^2 \varphi_o} \qquad \text{(VII-4)}$$

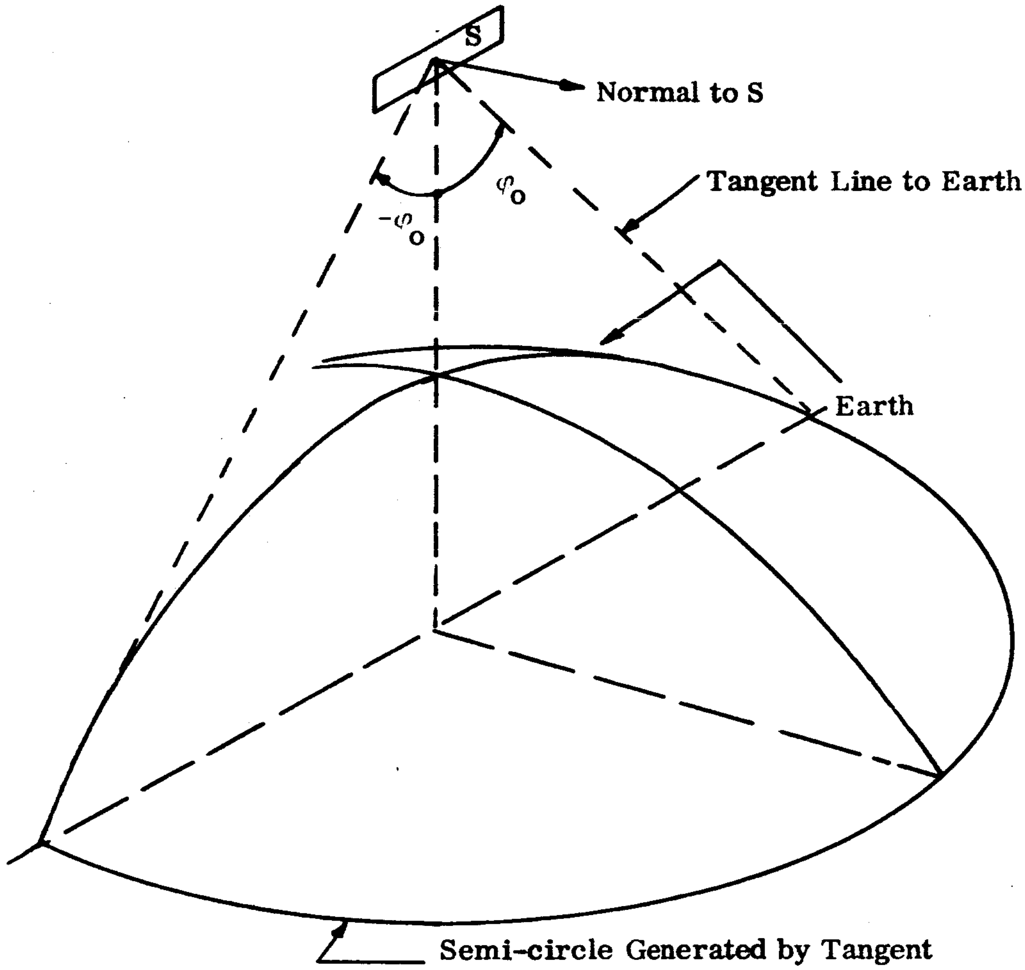
And the integral becomes

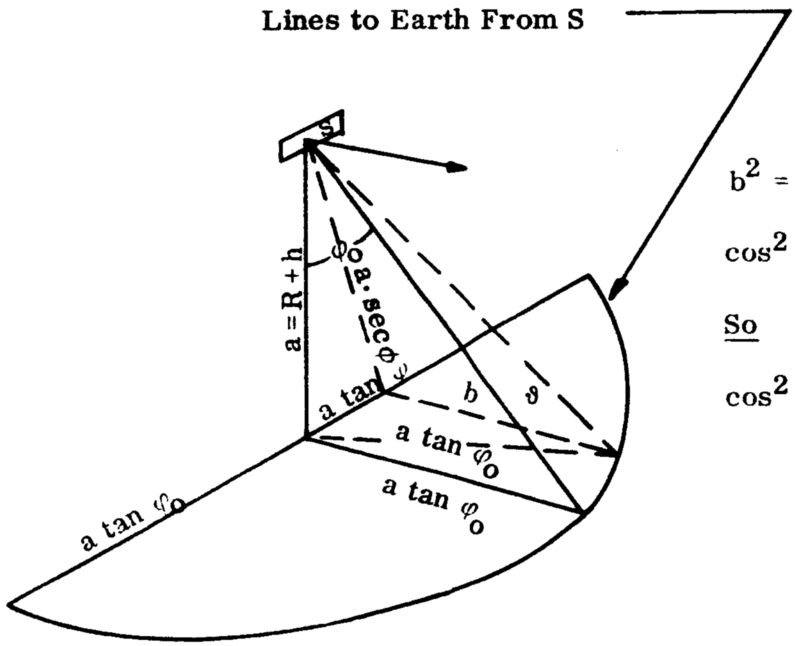
$$F_{se} = \frac{1}{2\pi} \left[2\varphi_o - \frac{1}{\sec^2 \varphi_o} \int_{-\varphi_o}^{\varphi_o} \sec^2 \varphi \, d\varphi\right] \qquad \text{(VII-5)}$$

$$F_{se} = \frac{1}{\pi} [\varphi_o - \sin \varphi_o \cdot \cos \varphi_o]$$

For $\varphi_o = 58.5$ degrees $= 1.0210$ radians, $\sin \varphi_o \cdot \cos \varphi_o = 0.4455$ and

$$F_{se} = \frac{0.5755}{\pi} = 0.1832 \qquad \text{(VII-6)}$$





$$b^2 = a^2(\tan^2 \varphi_o - \tan^2 \varphi)$$

$$\cos^2 \vartheta = \frac{b^2}{b^2 + a^2 \sec^2 \varphi}$$

<u>So</u>

$$\cos^2 \vartheta = \frac{\tan^2 \varphi_o - \tan^2 \varphi}{\tan^2 \varphi_o - \tan^2 \varphi + \sec^2 \varphi}$$

Figure VII-I. Geometry and Coordinate System

APPENDIX VIII

EFFECTIVE CONE EXTERNAL EMISSIVITY

The cone views the external environment by way of its mouth. Thus, from the point of view of the cone, the external environment may be replaced by an equivalent area stretched tightly over the cone mouth. In a manner similar to that used to derive the expression for radiant power transfer from the cone walls to the black patch (equation II-7 in Appendix II), one may derive an equation for the radiant transfer from the cone walls to the cone mouth (external environment). The result is

$$\Phi_{c-m} = \sigma T_c^4 \sum_{n=0} A_n' g_n' \left[1 - (1 - \epsilon_g)^n\right] \qquad \text{(VIII-1)}$$

where

σ = Stefan-Boltzmann constant

T_c = absolute temperature of the cone

ϵ_g = true surface emissivity of the cone

g_n' = view factor from A_n' to the cone mouth area A_m

A_n' is the cone wall area last intercepted by rays from the cone that require n-1 reflections at the cone wall to go out the cone mouth. Figure VIII-1 shows the A_n' areas for the vertical section of the first-stage cone and for rays which exit through the center of the cone mouth. Note that A_1' and A_2' are divided into sections separated on the cone surface.

But (M. Jakob, "Heat Transfer" Volume II, Wiley, 1957, p. 9)

$$A_n' g_n' = A_m f_n' \qquad \text{(VIII-2)}$$

where f_n' is the view factor from the cone mouth to the area A_n'. The view factor f_n' is also equal to the fraction of radiation entering the mouth of a perfectly reflecting cone that requires n cone reflections to go back out the cone mouth or to the black patch at the other end of the truncated cone. Substituting (VIII-2) into (VIII-1)

$$\begin{aligned} \Phi_{c-m} &= \sigma T_c^4 A_m \sum_{n=0} f_n' \left[1 - (1 - \epsilon_g)^n\right] \\ \Phi_{c-m} &= \sigma T_c^4 A_m \left\{\sum_{n=0} f_n' - \sum_{n=0} f_n' (1 - \epsilon_g)^n\right\} \end{aligned} \qquad \text{(VIII-3)}$$

But (M. Jakob, op. cit., p. 10)

$$\sum_{n=0} f_n' = 1 \qquad \text{(VIII-4)}$$

That is, all radiation which enters the cone mouth is reflected either to the patch (including the case of zero reflections) or back out the mouth in a perfectly reflecting cone. Or, in terms of the view factor, all radiation entering the mouth initially strikes either the cone wall or the black patch (for which n = 0 and $1-(1-\epsilon_g)^n = 0$).

Combining (VIII-3) and (VIII-4), the equation for the radiant flux emitted to the outside by the cone becomes

$$\Phi_{c-m} = \sigma T_c^4 A_m \left(1 - \sum_{n=0} f_n' \rho_g^n\right) \qquad \text{(VIII-5)}$$

where $\rho_g = 1 - \epsilon_g$ is the surface reflectivity of the cone. This may be written

$$\Phi_{c-m} = \sigma T_c^4 A_m (1 - \rho_{mx}) \qquad \text{(VIII-6)}$$

where ρ_{mx} is the effective or cavity reflectivity for radiation entering the cone mouth and equals the fraction of radiation incident on the mouth that is reflected back out the mouth and goes to the patch directly or by reflection. That is, it is the fraction entering that goes out both ends of the truncated cone cavity. The factor $1 - \rho_{mx}$ is the effective emissivity or absorptivity, ϵ_{mx}, as generally applied to cavities.

One may then define an effective cone external emissivity by the equation

$$\Phi_{c-m} = \sigma T_c^4 A_m \epsilon_{mx} = \sigma T_c^4 A_c \epsilon_{cx} \qquad \text{(VIII-7)}$$

where $A_c = \sum_{n=1} A_n'$ is the cone wall area.

Or

$$\epsilon_{cx} = \frac{A_m}{A_c} \epsilon_{mx} = \frac{A_m}{A_c} \left[1 - \sum_{n=0} f_n' (1 - \epsilon_g)^n\right] \qquad \text{(VIII-8)}$$

The cone external emissivity, ϵ_{cx}, is equal to the absorptivity of the cone walls for incident diffuse radiation (i.e., a source stretched tightly across the cone mouth), according to Kirchhoff's laws of radiation.

APPENDIX IX

EFFECTIVE CONE MOUTH ABSORPTIVITY FOR EARTH RADIATION

The view to earth from the mouth of the cone is the same as its view to a vertical semicircle whose radius subtends an angle equal to the angle from the spacecraft nadir to a tangent line to the earth and whose center lies on the nadir (See Appendix VII). If we set up spherical coordinates at the cone mouth, with the pole along a normal to the mouth, the number of reflections earth radiation undergoes at the cone walls before going back out the cone mouth is a step-wise function of the polar angle ϑ (See Figure IX-1) for a right circular cooler cone. That is, the range of polar angles can be divided into regions in which the absorptivity in the cone walls is a constant. Alternatively, the surface area of the earth seen from the cone mouth, or the equivalent area of the semicircle, can be divided into sub-areas, A_{en}, whose radiation is reflected n times at the cone walls upon entering the cone mouth. The effective absorptivity of the cone mouth for earth radiation is then

$$a_{me} = \Sigma\, a_n\, F_m\text{-}A_{en} \qquad \text{(IX-1)}$$

where a_n is the absorptivity produced by n cone-wall reflections and F_m-A_{en} is the view factor from the cone mouth to the sub-area A_{en}.

For the spherical coordinate system shown in Figure IX-1, the view factor is given by

$$F_m\text{-}A_{en} = \frac{2}{\pi} \int_{o}^{\varphi_n} \int_{\vartheta_n}^{\vartheta_{n-1}} \sin\vartheta \cos\vartheta \; d\vartheta\, d\varphi + \frac{2}{\pi} \int_{\varphi_n}^{\varphi_{n-1}} \int_{\vartheta(\varphi)}^{\vartheta_{n-1}} \sin\vartheta \cos\vartheta \; d\vartheta\, d\varphi \qquad \text{(IX-2)}$$

Integrating the first term fully and the second term with respect to the polar angle, the view factor becomes

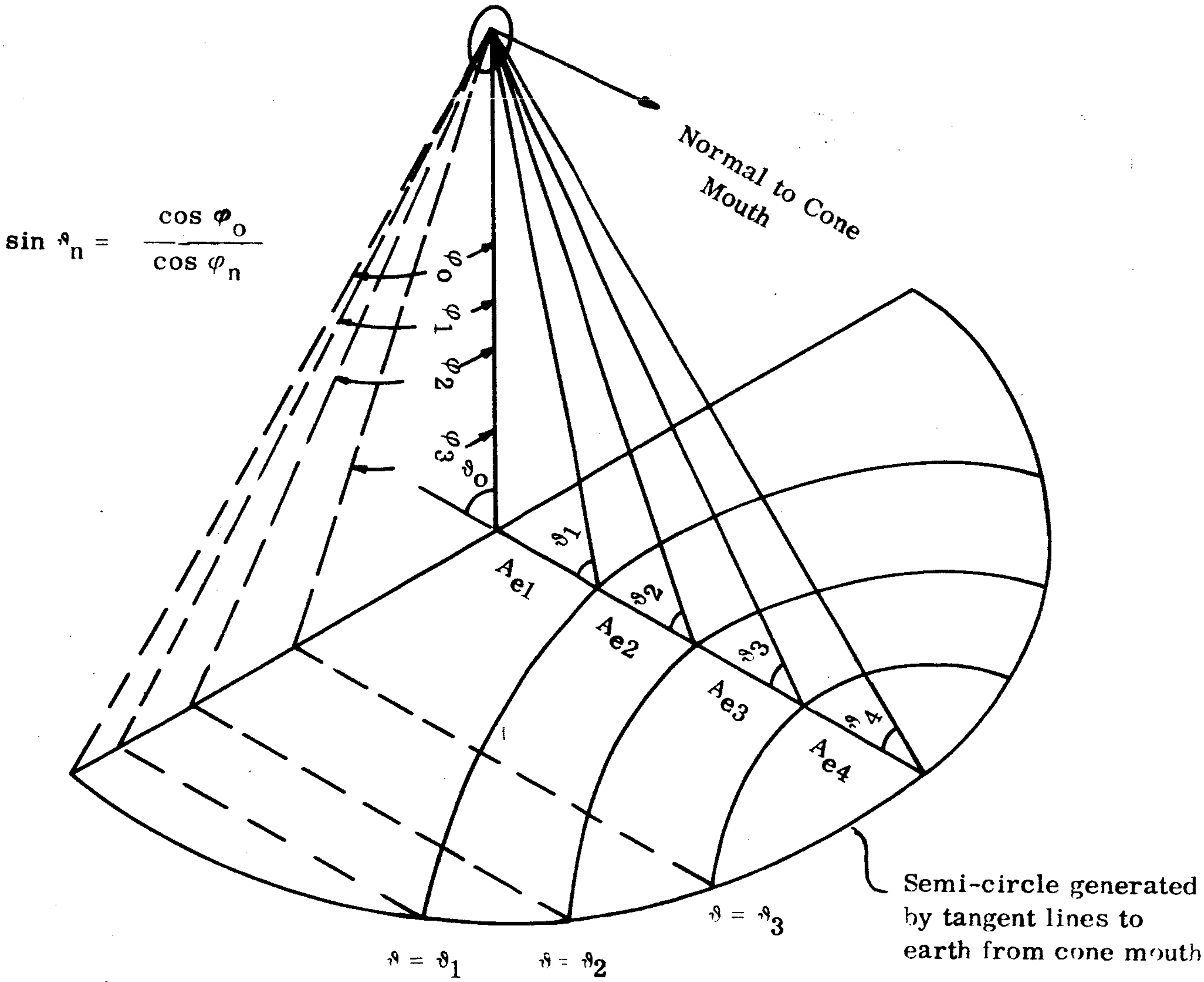
$$F_m\text{-}A_{en} = \frac{1}{\pi}\, \varphi_n \left(\sin^2 \vartheta_{n-1} - \sin^2 \vartheta_n\right) + \frac{1}{\pi} \int_{\varphi_n}^{\varphi_{n-1}} \left(\sin^2\vartheta_{n-1} - \sin^2 \vartheta(\varphi)\right) d\varphi$$

But, from Appendix VII

$$\sin^2 \vartheta(\varphi) = \frac{\sec^2 \varphi}{\sec^2 \varphi_o} \qquad \text{(IX-3)}$$

and

$$\sin^2 \vartheta_n = \frac{\sec^2 \varphi_n}{\sec^2 \varphi_o} \qquad \text{(IX-4)}$$



$\vartheta_o = \pi/2$

$\vartheta_4 = \pi/2 - \varphi_o$

$\varphi_4 = 0$

Surfaces ϑ = constant are cones with axes along normal to cone mouth and intercept semi-circle in hyperbolae.

Figure IX-1 Coordinates and Angles for Calculation of Cone Mouth Absorptivity For Earth Radiation

Thus

$$F_{m}\text{-}A_{en} = \frac{1}{\pi \sec^2 \varphi_o} \left[\varphi_{n-1} \sec^2 \varphi_{n-1} - \tan \varphi_{n-1} - (\varphi_n \sec^2 \varphi_n - \tan \varphi_n)\right] \tag{IX-5}$$

Combining equations (IX-1) and (IX-5), we obtain

$$a_{me} = \frac{1}{\pi \sec^2 \varphi_o} \sum_{n=0}^{\rho} (a_{n+1} - a_n)(\varphi_n \sec^2 \varphi_n - \tan \varphi_n) \tag{IX-6}$$

where $\rho + 1$ is the maximum number of cone-wall reflections. For the special case $\rho = 0$ (i.e., a maximum of one reflection),

$$a_{me}\ (\rho = 0) = a_1 \frac{(\varphi_o - \sin \varphi_o \cdot \cos \varphi_o)}{\pi} = a_1 F_{me} \tag{IX-7}$$

where a_1 is the absorptivity for a single cone reflection and F_{me} is the view factor from the cone mouth (an outside vertical cooler surface) to the earth (See Appendix VII).

APPENDIX X

TEMPERATURE OF COMBINED CONE AND OUTER SURFACE

For sufficiently low values (0.04 or less) of gold emissivity, the first-stage cone is to be thermally tied to the outer surface of the cooler. In this case the cone absorbs a net radiant power that is a very small fraction of the power radiated by the outer surface. The combined cone and outer surface then assume the temperature of the outer surface when it is not thermally tied to the cone.

Using the notation in Section 4.2, the net power absorbed per unit area of the first-stage cone is (for T_{p1}^4 much less than T_c^4)

$$\frac{\Delta\Phi c}{Ac} = \alpha_{ce} W_s + \epsilon_{ce} W_e - [\epsilon_{cx} + \epsilon_{cp} (1 + 1/2 \frac{Ac2}{Ap1})] \sigma T_c^4 \qquad \text{(X-1)}$$

For (See Table 5, Section 4.2.2)

$$\epsilon_g = 0.02$$

$$\alpha_g = 0.183$$

$$T_c = 248^o K$$

$$1/2 \frac{Ac2}{Ap1} = 0.492$$

We obtain

$$\frac{\Delta\Phi c}{Ac} = 1.96 \times 10^{-4} \text{ watts/cm}^2 \qquad \text{(X-2)}$$

The net radiant power absorbed by the cone adds a term

$$\frac{\Delta\Phi c}{Ac} \times \frac{Ac}{A_3 \epsilon_2} = 1.03 \times 10^{-3} \text{ watts/cm}^2 \qquad \text{(X-3)}$$

for Ac/A3 = 4.75 and ϵ_2 = 0.9 to the right side equation (10) in Section 4.1. The net radiant power absorbed by the cone is therefore equivalent to increasing the radiant emittance of the outer surface by

$$\Delta \sigma T_s^4 = \frac{1.03 \times 10^{-3}}{4.51} = 2.28 \times 10^{-4} \text{ watts/cm}^2 \qquad \text{(X-4)}$$

This is about 1 percent of σT_s^4 and would increase T_s by about 1/4 percent or 0.7 degree K, which is much less than the uncertainty in temperature of the outer surface. When the first-stage cone is thermally connected to the outer surface, it therefore assumes the temperature of the outer surface.

APPENDIX XI

CONE MOUTH TO EARTH VIEW FACTORS WITH EARTH SHIELD IN PLACE

The view factors from the cone mouth to the earth in the absence of an earth shield are determined in Appendix IX. The surface area of the earth, or its equivalent semicircle, was divided into sub-areas, A_{en}, whose radiation is reflected n times in the cone. The absorptivity, a_n, is then constant for radiation from a given sub-area, and the effective absorptivity of the cone mouth for radiation from the entire surface area of the earth is given by

$$a_{me} = \Sigma\, a_n F_{m}\text{-}A_{en} \qquad \text{(XI-1)}$$

where F_m-A_{en} is the view factor from the cone mouth to the sub-area A_{en}.

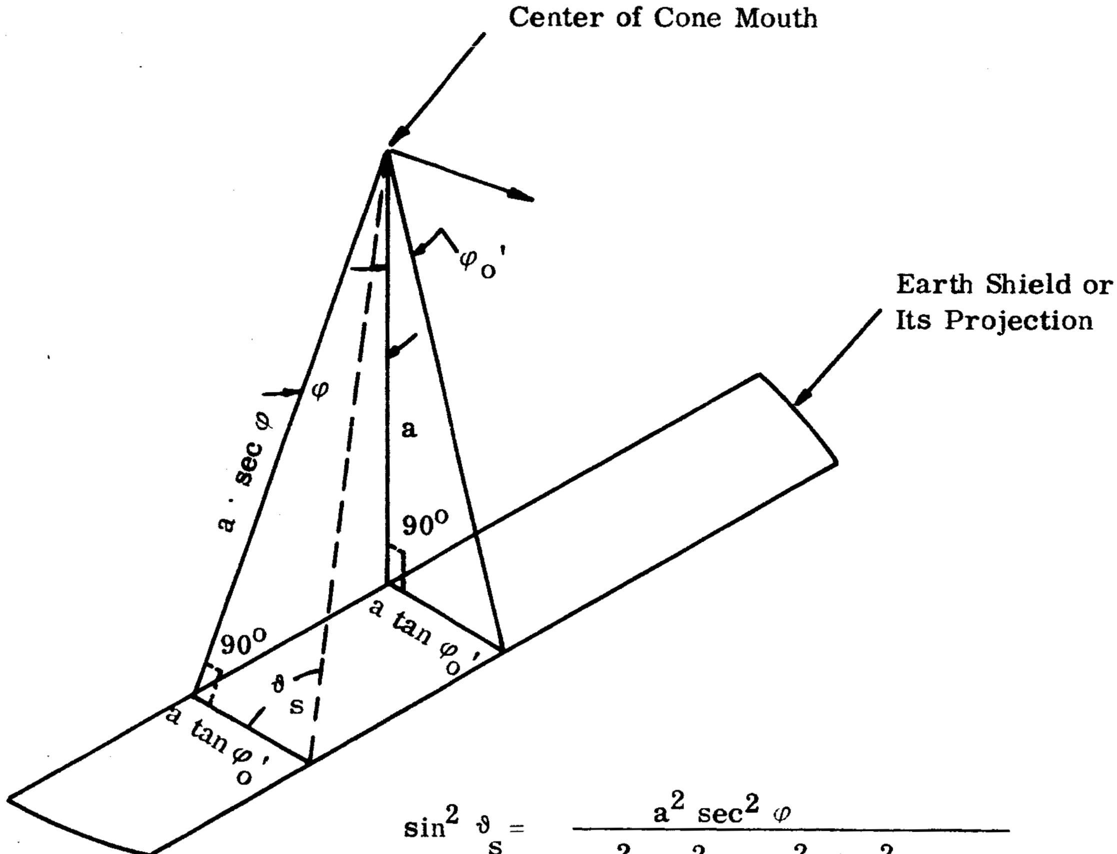
The earth shield shown in Figure XI-1 was designed to block all radiation to the cone mouth from the sub-area A_1 and part of the radiation from A_2. No radiation from A_3 and A_4 is blocked. This is shown in Figure XI-1 by the shadow cast unto the semicircle (equivalent earth) by the shield from the center of the cone mouth. With the shield in place the view factor for n = 1 is therefore zero, while it is unchanged for n = 3 and n = 4. It therefore remains to calculate the view factor for n = 2. For n = 2 the earth shield changes the upper limit of the polar angle in the second equation in Appendix IX. This equation gives the view factor, F_m-A_{en}, in terms of spherical coordinates at the center of the cone mouth, with the pole along the normal to the mouth. The polar angle is ϑ and the azimuthal angle φ. With the shield in place the upper limit of the polar angle for n = 2 is not ϑ_1 but the angle, ϑ_s, determined by the straight line projected across the semi-circle (See Figure XI-1).

The view factor from the cone mouth to the visible part of earth whose radiation is reflected twice in the cone is then

$$F^{\text{vis.}}_{m\text{-}Ae_2} = \frac{2}{\pi}\int_0^{\varphi_2}\int_{\vartheta_2}^{\vartheta_s} \sin\vartheta\cos\vartheta\, d\vartheta\, d\varphi + \frac{2}{\pi}\int_{\varphi_2}^{\varphi_1}\int_{\vartheta(\varphi)}^{\vartheta_s} \sin\vartheta\cos\vartheta\, d\vartheta\, d\varphi \qquad \text{(XI-2)}$$

Integrating with respect to the polar angle,

$$F^{\text{vis.}}_{m\text{-}Ae_2} = \frac{1}{\pi}\int_0^{\varphi_2} (\sin^2\vartheta_s - \sin^2\vartheta_2)\, d\varphi + \frac{1}{\pi}\int_{\varphi_2}^{\varphi_1} (\sin^2\vartheta_s - \sin^2\vartheta(\varphi))\, d\varphi \qquad \text{(XI-3)}$$



$$\sin^2 \vartheta_s = \frac{a^2 \sec^2 \varphi}{a^2 \sec^2 \varphi + a^2 \tan^2 \varphi_o'}$$

$$\sin^2 \vartheta_s = \frac{1}{1 + \cos^2 \varphi \cdot \tan^2 \varphi_o'}$$

Figure XI-1 Relation Between ϑ and φ along straight edge of earth shield or its projection.

The equation for $\sin^2 \vartheta\ (\varphi)$ is derived in Appendix VI. The result is

$$\sin^2 \vartheta\ (\varphi) = \frac{\sec^2 \varphi}{\sec 2\ \varphi_o} \qquad \text{(XI-4)}$$

This is the relation between ϑ and φ along the semicircle generated by tangent lines to the earth from the cone mouth.

Along the straight line projection on the semicircle (See Figure XI-1)

$$\sin^2 \vartheta_s = \frac{1}{1 + \cos^2 \varphi \cdot \tan^2 \varphi_o'} \qquad \text{(XI-5)}$$

where φ_o' is the vertical-plane angle subtended by the earth shield at the center of the cone mouth (31.5 degrees in Figure XI-1). The earth shield was designed for the straight line to intersect the semicircle at $\vartheta = \vartheta_1 = 71$ degrees 27 minutes (See Figure IX-1 in Appendix IX). Since $\varphi_o = 58.5$ degrees is the angle between the nadir and the tangent line to the earth, equations (XI-4) and (XI-5) give

$$\tan \varphi_o' = \frac{\cos 71^o\ 27'}{\cos 58.5^o} = 0.61 \qquad \text{(XI-6)}$$

and φ_o' is approximately 31.5 degrees.

Substituting equations (XI-4) and (XI-5) into (XI-3), we obtain

$$F^{vis.}_{m-Ae_2} = \frac{1}{\pi} \int_0^{\varphi_1} \frac{d\varphi}{1 + \cos^2 \varphi \cdot \tan^2 \varphi_o'} - \frac{\varphi_2}{\pi} \sin^2 \vartheta_2$$

$$- \frac{1}{\pi} \int_{\varphi_2}^{\varphi_1} \frac{\sec^2 \varphi\, d\phi}{\sec^2 \varphi_o} \qquad \text{(XI-7)}$$

Integrating (the first integral is given by G. Petit Bois, "Tables of Indefinite Integrals", Dover, 1961, p. 122) and rearranging terms

$$F^{vis.}_{m-Ae_2} = \frac{1}{\pi} [\cos \varphi_o' \cdot \arctan (\cos \varphi_o' \cdot \tan \varphi_1)$$

$$- \cos^2 \varphi_o\ (\varphi_2 \sec^2 \varphi_2 + \tan \varphi_1 - \tan \varphi_2)] \qquad \text{(XI-8)}$$

For φ_o' = 31.5 degrees

φ_o = 58.5 degrees

φ_1 = 56 degrees 33 minutes

φ_2 = 51 degrees 45 minutes,

$$F^{\text{vis.}}_{m-Ae_2} = 0.02085 \qquad \text{(XI-9)}$$

The values given for φ_1 and φ_2 are for a truncated right circular cone having the geometry of the vertical cooler plane.

APPENDIX XII

USE OF HELIUM REFRIGERATOR TO COOL COPPER COLD REFERENCE

Based on Calculations

by

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The following calculations cover the application of a Norelco A-20 helium refrigerator to the cooler test program (Section 5.1). The calculations show that it is feasible to use the A-20 with a solid copper shield (Figure XII-1) attached to its cold head within the space chamber to conduct the heat away from the surface viewed by the radiant cooler.

The total delta T across the copper sideplates is approximately 2 degrees K, based on a copper sheet 1/2 cm thick. If we can increase this thickness to 1 cm, we can effectively halve the delta T. We have also calculated the temperature drop across the conducting bars to the A-20 head assuming the copper bars are 2.5 cm thick and 10 cm wide. This temperature difference is approximately 10 degrees K. Again we can help ourselves by increasing the conductive cross-sectional area. However, the weight of the set-up is already about 120 pounds, which will require some internal support of the copper shield.

It is interesting to note that, based on available data, there are several coppers which have a higher thermal conductivity than OFHC copper. The high purity annealed copper has a conductivity five times higher and would reduce the total weight by 75 to 80 percent. To date we have had very little success in determining the availability of this high purity copper.

The 77 degrees K shroud around the outside of the copper shield can be cooled by attachment to the first stage of the A-20 refrigerator.

A somewhat simplified calculation of the temperature distribution in the shield can be made following some reasonable assumptions:

1. The temperature of the A-20 refrigerator lead is ~25 degrees K;

2. The heat flux on the side plates is uniform;

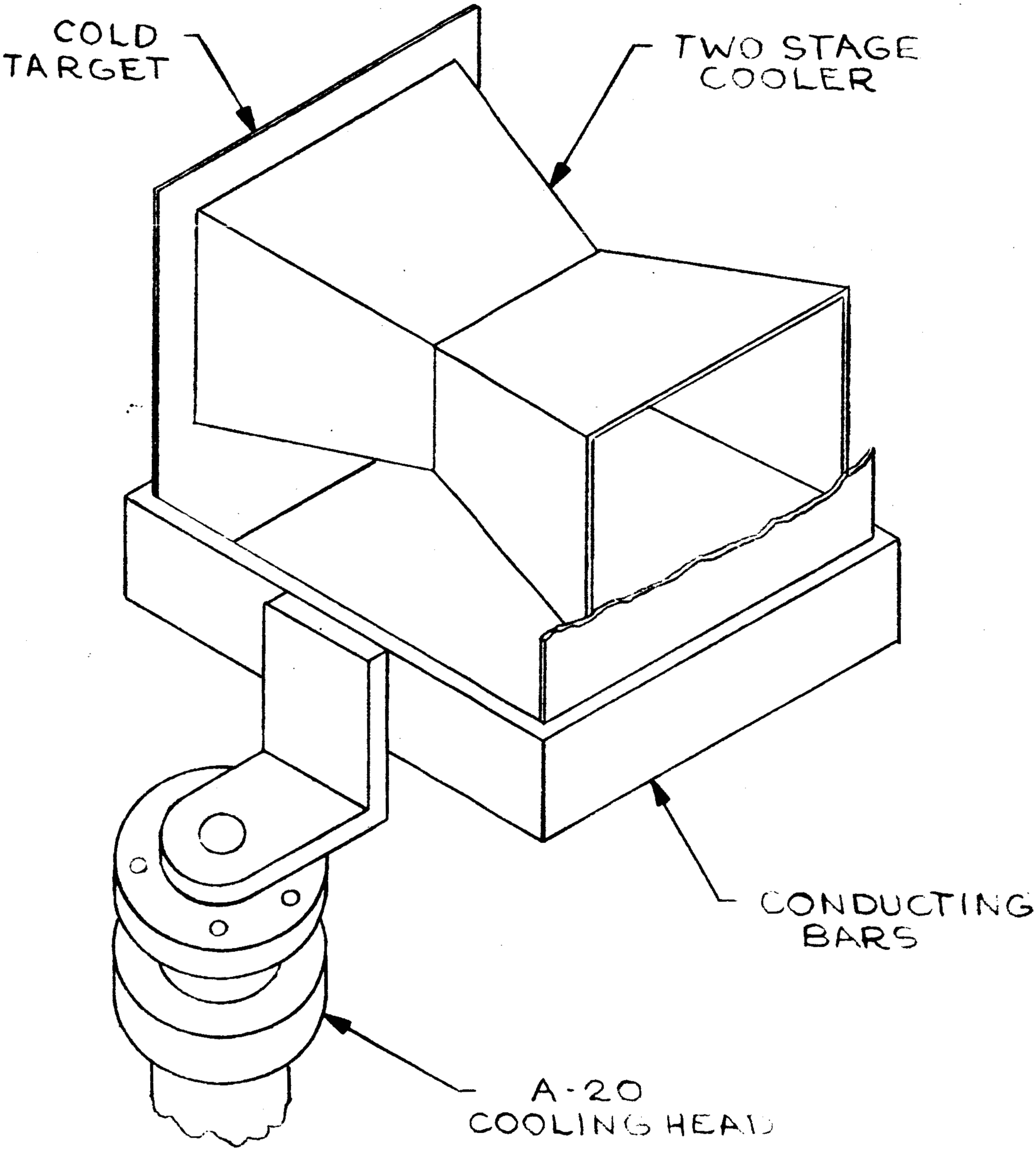
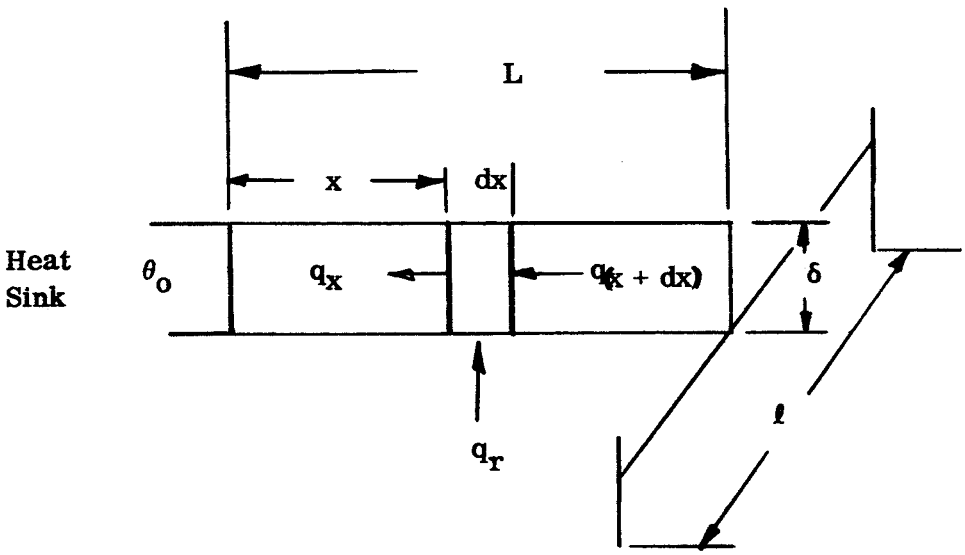


Figure XII-1 Attachment of Cold Reference to Helium Refrigerator

3. The heat flux (from test object) is ~30 watts/plate

4. Shielding around the complete assembly is maintained at 77 degrees K so that the radiative transfer contribution from the surroundings is negligible.

Consider now a plate of thickness δ, width ℓ, and length L attached to a heat sink at temperature θ_0: [see P. J. Schneider, "Conduction Heat Transfer," Addison-Wesley Publ. Co., Cambridge, Mass. (1955) p. 36]



The element dx of the plate conducts longitudinal heat q_x and $q_{(x+dx)}$ and gains lateral heat q_r by radiation. If the temperature at a distance x from the heat sink is θ, the temperature at x+dx will be greater than θ by about $-(\frac{d\theta}{dx})dx$. The steady-state heat balance is then

$$q_r + q_{(x+dx)} - q_x = 0$$

In terms of differentials this becomes

$$q_r + [-K A \frac{d}{dx} (\theta - \frac{d\theta}{dx})dx] -[-K A \frac{d\theta}{dx}] = 0$$

or

$$q_r + K A \frac{d^2\theta}{dx^2} dx = 0$$

Making use of assumption 2 above, it is assumed that the radiation flux is uniform over (one side of) the plate. If the rate is λ watts per cm^2, q_r is given by

$$q_r = \lambda \ell \, dx.$$

The differential equation becomes

$$\lambda \ell + K A \frac{d^2\theta}{dx^2} = 0$$

or

$$\frac{d^2\theta}{dx^2} = -\frac{\lambda \ell}{K A} = -c$$

where $c = \frac{\lambda \ell}{K A}$ is a positive constant.

This equation is satisfied by a solution of the form

$$\theta_x = \alpha x^2 + \beta x + \gamma$$

where α, β, γ are constants to be evaluated by applying the appropriate boundary conditions.

Taking the derivatives we find

$$\alpha = \frac{-c}{2}$$

next we apply the condition

$$\frac{d\theta}{dx} = 0 \quad \text{at } x = L$$

This implies that there is no flow of heat past the outer edge of the plate, i.e.

$$q_L = -K A \left(\frac{d\theta}{dx}\right)_L = 0$$

$$\left.\frac{d\theta}{dx}\right)_L = 2 \alpha L + \beta = 0$$

$$\therefore \beta = cL$$

The remaining boundary condition is

$$\theta_x)_o = \theta_o$$

$$\therefore \gamma = \theta_o$$

The temperature distribution is given by

$$\theta_x = \frac{-c}{2} x^2 + cL x + \theta_o$$

The temperature difference is thus

$$\theta_x - \theta_o = \frac{-c}{2} x^2 + cL x.$$

At x=L

$$\theta_L - \theta_o = \frac{-c}{2} L^2 + cL^2 = \frac{cL^2}{2}$$

The constant c can be evaluated from the condition that the total radiation Q on the plate is given by

$$Q = \int_{x=0}^{L} \lambda \ell \, dx = \lambda \ell L$$

so $\lambda \ell = \frac{Q}{L}$.

The following numerical values apply:

ℓ = 50 cm
L = 40 cm
δ = 0.5 cm
Q = 30 watts
K = 12 watts/cm^{o}K for OFHC Copper
14 watts/cm^{o}K for Electrolytic Tough Pitch Copper
23 watts/cm^{o}K for Coalesced Copper
60 watts/cm^{o}K for Hi-Purity Annealed
A = $\ell \delta$

Using the value for OFHC copper

$$c = \frac{\lambda \ell}{KA} = \frac{30/40}{12 \times 50 \times 0.5} = \frac{1}{400} = .0025 \text{ deg K/cm}^2$$

The temperature drop across the plate is then

$$\theta_L - \theta_o = \frac{cL^2}{2} = \frac{0.0025}{2} \times 1600 = 2^o \text{ K}$$

A similar calculation assuming all the heat into the top end of the plate (at x=L) gives a temperature difference of 4.0 degrees K which is still within limits.

Considering the bottom crossbar which supports the end plates under the assumption of uniform loading along the length of the bar as in the above calculation, the temperature difference amounts to 2.5 degrees K. (In this case the cross sectional area is 10 cm x 2.5 cm and the length is 50 cm).

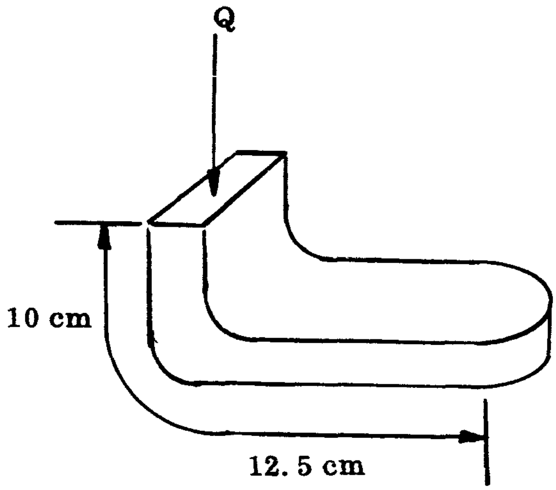
The crossbar supporting the end plates is considered as if all the heat flux from the end bars is conducted through the cross sectional area of the bar. The defining equation for heat conduction can be applied, viz.

$$\Delta \theta = \frac{QL}{KA}$$

where symmetry conditions apply

$$\Delta\theta = \frac{30 \times 23}{12 \times 25} = 2.3^{o}K$$

A similar consideration may be applied to the supporting angle piece



Assuming a total length L=23 cm, total load = 60 watts

$$\Delta\theta = \frac{60 \times 23}{12 \times 25} = 4.6^{o}K$$

The contacting interfaces may also show a temperature drop. Based on experimental data for various fillers, we may apply

$$\Delta\theta = \frac{1}{h_c} \times \frac{Q}{\text{Contact Area}}$$

The quantity $1/h_c$ varies from 1.5 for bare contact to 0.3 for glycerol + silver powder while indium foil has $1/h_c \approx 1.0$. At the flat surface/conducting bar Q=30 watts, Contact Area=50 cm x 10 cm

$$\Delta\theta = 1 \times 30/500 \approx 0.06^{o}K$$

Across conducting bar and crosspiece:

$$\Delta\theta = 1.0 \times 60/100 \approx 0.6^{o}K$$

Crosspiece to cold lead of A-20

$$\Delta\theta = 1.0 \times 60/80 = 0.75^{o}K$$

The total $\Delta\theta$ from plates to A-20 lead is then $2.5 + 2.3 = 4.8^{o}K$ at conducting bar

$$
\begin{array}{rll}
2.5 + 2.3 = & 4.8^{o}K & \text{at conducting bar} \\
 & 4.6^{o}K & \text{at crosspiece} \\
.6 + .75 + .06 = & \underline{1.4^{o}K} & \text{at interfaces} \\
\text{total} & 10.8^{o}K &
\end{array}
$$

The A-20 lead will operate at approximately 17 degrees K so the temperature in the warmest part of the end plates is ~29 to 30 degrees K.

Doubling the thickness of copper would reduce the $\Delta\theta$'s by ~1/2, and further improvement could be expected if the high purity copper can be employed.

The total amount of copper is:

Plates	2 x 0.5 x 40 x 50	= 2000 cm^3
Bars	(50 + 50 + 55) x 25 x 10	= 3900 cm^3
	(23) x 2.5 x 10	= 600 cm^3
		6500 cm^3
Weight	9 gr x 6500 = 58,500 grams	
	~120 lbs.	